NATIONAL SPORTS ACADEMY “VASSIL LEVSKI”

Department “Gymnastics”

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BIOMECHANICAL MODELS OF NON-SUPPORT PHASE OF GYMNASTICS Elements WITH COMBINATIONS OF ROTATIONS

AUTHOR’S SUMMARY

of dissertation

for acquiring a scientific degree

„Doctor of Science” in professional field 7.6. Sport

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Sofia, 2021

The dissertation was discussed during an internal defense on 19.10.2021 and an official defense was appointed by Department “Gymnastics” at National Sports Academy “Vassil Levski”.

The dissertation consists of 314 pages, 104 figures, 7 appendices. The reference list includes 237 sources, of which 75 in Cyrillic and 162 in Latin.

The public defense of the dissertation will take place on 22.02.2022 at 2 p.m. at A-3 Hall at NSA “Vassil Levski”, Sofia 1700, Students Town, 21 “Academic Stefan Mladenov” str.

Introduction

The perfection of sports technique and the increase in the efficiency of motor activities require the successful solving of a number of issues related to movement control.

To be useful enough for the practice, sports science should serve not only for analyses of existing real movements but also for finding new motor programs with desired characteristics with proven efficiency which can be executed at ease. Sport, like any other scientific field, is known for the primary role of mathematical modelling for performing efficient cognitive activities and finding certain motor optimality. This is especially true of gymnastics – a sport with a wide varierty of motor forms and innumerable movements where a number of issues are waiting for their answers, and the problems related to technique are waiting for their successful solution.

Gymnastics exercises consist of interrelated phases, each of which is characterised with its own technical peculiarities and control mechanisms. Although the main characteristics of the non-support (aerial) phase are determined during the support period (the trajectory of center of gravity, angular momentum, fly time), in the flight there are significant possibilities for influence and control on the movement (the rotation component). This can be mostly applied to non-support phases with combinations of rotations (e.g., a kind of somersault with twist around the longitudinal axis). The motor behaviour of the body during the non-support phase is determined by specific biomechanical principles, and the quality of execution of this phase can be taken as an indicator for the technical (motor) capacity of the executor. In most cases, the non-support phase is the most attractive part of the movement which makes athletes direct their efforts to perfecting the characteristics of the flight phases.

The traditionally applied approach, where the conclusions are drawn on the basis of certain real performances, will lead to restrictic knowledge related only to the researched performances, i.e., the scope of the summary will be restricted only to the area of the existing motor manifestations. This means that each perfection of the technique will follow only the practical experience and real-time realizations. Seeking new efficient technical solutions would be based only on logical assumptions or coaches and scientists' intuition and the introduction and experimentation in practice of motor novelties without clarified quantitative characteristics and lack of clear vision of the expected results. Besides a waste of time and efforts, in some cases it can hide quite a risk of injuries.

With the introduction of mathematical modelling and computer technologies in the intstruments of the researchers when surveying the movement, the cognitive productivity of the presented scientific work has increased significantly. In the purposeful control of the conditions and the managing parameters and variables, we can significantly increase our research efficiency in finding a certain motor optimality. Performing different manipulations with the managing parameters of the model, we can ensure a certain behaviour which is difficult to be achieved (or impossible) when conducting practical experiments in real time. Apart from that, we can easily determine the leading factors in a certain motor manifestation, as well as establish the sensitivity of initial conditions and different structural components on the motor behaviour of the body of an athlete. Besides being a powerful instrument for a fast finding and evaluating the efficient technical solutions, modelling can be also applied in creating new exercises.

Regardless of the indisputable achievements related to numerous gymnastics exercises, there are not detailed programs ensuring the successful execution of non-support phases of the exercises. A number of specific questions related to productivity of various motor actions and applied strategies are also waiting for satisfactory answers. This motivated us, after taking advantage of the great possibilities modelling provides, to clarify both the factors responsible for the efficiency of actions and the possibilities for control of the movements in the context of a particular motor program related to different basic exercises and different apparatus. We believe that for a sport such as gymnastics it is necessary that we develop models for efficient motor behaviour related to a number of particular exercises on different apparatus. We should create motor programs ensuring the transition of a motor habit to more complex variations of certain exercises. This means that we should achieve both technical optimality and methodological rationality.

On the basis of the analysis of the literary sources and the results from the preliminary numerical experiments, we created ***<u>the hypothesis of the research</u>***:

We assume that we can create reliable, methodologically grounded, technically rational models of the non-support phase of some basic gymnastics elements with combinations of rotations. The modelled movements can be shown with a sequence of images, supplemented by descriptions, explanations related to the technical construction and quantitative parameters of the exercises. We believe that the creation and introduction of sample performances of different types of motor behaviour are a prerequisite for both increasing the efficiency in working for high technical mastery and enriching sports pedagogues' theoretical knowledge about control mechanisms and factors predetermining motor efficiency during the non-support phase.

2 Aim, tasks, organization and methods of research

2.1 Aim of the research:

The aim of the research was to establish rational motor actions which can be diferentiated as sample performances of gymnastics elements with combinations of rotations, and the applied motor strategies should allow an easily acceccible transition to the higher levels of difficulty of the elements.

2.2 Tasks of the research:

1. Observation of selected video materials of the motor activity and the applied motor strategies in performing gymnastics elements with combinations of rotations during the non-support phase;
2. Development of a mathematical model of the body of an athlete with a high degree of agility for computer simulations of spatial movements during the non-support phase of exercises;
3. Establishemnt of objective mechanical effects in the behaviour of the system of connected bodies (manifested in non-support state) and combining certain effects with gymnastics technique;
4. Development of rational technical solutions in the context of particular gymnastics elements;
5. Development of motor strategies for initiation of twists along the longitudinal axis on the basis of motor resemblance between the different levels of complexity of the elements from a certain type;
6. Development of sample performances of gymnastics elements to be used as a technical training model which can be successfully made more complicated.

2.3 Organization of the research:

I stage (2012 - 2018):

– Review of major literary sources, analysis, and summary - 2012 - 2018;

– Review and selection of video materials - 2014 - 2018;

– Development of a mathematical model for computer simulations (working out the equations of the motion and transforming them into a kind suitable for direct compuetr realization, development of the main program for conducting simulations, development of subprograms for additional calculations; validation of the model) - 2012- 2018;

<u>II stage</u> (2016 - 2020):

– Conducting numerical experiements - 2016 - 2019:
– Making sample performances of basic gymnastics elements - 2017 - 2019:
– Writing the dissertation and preparing for internal defense - 2021

Object of research:

Object of research is basic gymnastics elements with combinations of rotations during the non-support phase.

Subject of the research:

Subject of the research is motor actions and motor strategies which are applied for initiation and termination of the twist along longitudinal axis during the non-support phase and the biomechanical characteristics preconditioning the movement during the non-support phase.

2.4 Research methods:

– Review, analysis, and summary of the literary sources;
– Video observation;
– Mathematical modelling;
– Numerical experiments;
– Biomechanical analysis.

2.4.1 Review, analysis and summary of the literary sources

We reviewed 237 literary sources, of which 75 in Cyrillic and 162 in Latin.

2.4.2 Video observation

We watched video materials including gymnastics routines from different competitions (Olympic Games, World and European Cups, International tournaments). We selected gymnastics elements (87 from male and female gymnastics) executed by different athletes. We applied computer program (Kinovea) for in-frame view of the motions in order to detect some details from the performance – initial configuration, motor strategies for different elements, duration of the non-support phase, duration of different motor actions.

2.4.3 Mathematical modelling

2.4.3.1 Structure of the model.

The presented model in this dissertation is angle-driven . The model consists of 16 rigid segments which include: three segments for each upper limb (upper arm, forearm, hand); three segments for each lower limb (thigh, shank, foot); one segment for head and neck. To achieve greater mobility, we divided the torso into three parts. The relative motions among the sixteen segments are helped by 15 joints allowing a very high degree of agility – 32 inner degrees of freedom. The outer degrees of freedom are 6 (3 rotational and 3 translational).

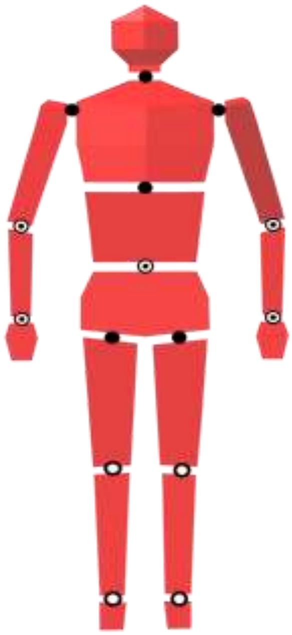

Fig. 2.1 Body segments and joints. The number of the degrees of freedom in the joints is indicated with: ⬣ - 3 degreees of freedom; ⦿ - 2 degrees of freedom; ○ - 1 degree of freedom

The characteristic anatomical movements accessible for immitation are: at the shoulder and hip joints – flexion/extension, abduction/adduction, internal/ external longitudinal rotation; at the elbow joints – flexion/extension, pronation/supination; at the wrist joints – flexion/extension and abduction/adduction; at the ankle joints – flexion/extension. The movements between the upper and middle part of the torso are: flexion/extension, left/right lateral flexion, longitudinal rotation. Between the middle and lower part of the torso – flexion/extension and left/right lateral flexion. The head has three rotation degrees of agility which allow for flexion/extension, lateral bending to the left/right and longitudinal rotation. The number of the rotations of each joint is shown in fig. 2.1.

2.4.3.2 Determining the anthropometric and inertia characteristics. Tensor of inertia

In order to determine the inertia characteristics of the body, we applied the method described by Zatsiorsky et al. (1981)[1]. To calculate the length of the segments, we used the relevant tables of coefficients of the equations of multiple regression of the kind $y = B_0 + B_1x_1 + B_2x_2 + B_3x_3$, at: leg length ($x_1$), body length ($x_2$) and arms' length ($x_3$). We also used equations of the kind $y = B_0 + B_1x_1 + B_2x_2$ for calculation of other inertia characteristics relevant to weight (x_1) and body length (x_2).

In rotations, an important inertia characteristic is the tensor of inertia of the body ($\mathbf{I}_c$). Presented in a matrix form, it is the following:

$$\mathbf{I}_\mathrm{c} = \begin{bmatrix} I_\mathrm{xx} & -I_\mathrm{x\mathit{y}} & -I_\mathrm{xz} \\ -I_\mathrm{yx} & I_\mathrm{yy} & -I_\mathrm{yz} \\ -I_\mathrm{zx} & -I_\mathrm{zy} & I_\mathrm{zz} \end{bmatrix}. \tag{2.1}$$

In the model, the axes of the local coordinate system of each segment ($C_i\, X_i\, Y_i\, Z_i$) are orientated in a way so that their direction coincides with the longitudinal, frontal, and sagittal axis of the particular segment, and their origin is in the local centers of mass. This means that in relation to its own local coordinate system, the te nsor of inertia of each segment is a kind of diagonal, and its diagonal elements are the inertia moments related to the principal axes of the segment. We calculated the values of these moments of inertia according to the method described by Zatsiorsky et al. (1981).

The tensor of inertia of the segment „ i “, defined in the coordinate system $CX_CY_CZ_C$ (fig. 2.2), is calculated with the expression:

$$\mathbf{I}_\mathrm{i}^\mathrm{c} = \mathbf{I}_\mathrm{tr}^\mathrm{c} + \mathbf{I}_\mathrm{rot}^\mathrm{c}\,,$$

$$\text{where} \quad \mathbf{I}_\mathrm{tr}^\mathrm{c} = \mathrm{m_i}.\begin{bmatrix} r_\mathrm{i\,y}^2 + r_\mathrm{i\,z}^2 & -r_\mathrm{ix}r_\mathrm{iy} & -r_\mathrm{ix}r_\mathrm{iz} \\ -r_\mathrm{iy}r_\mathrm{ix} & r_\mathrm{i\,x}^2 + r_\mathrm{i\,z}^2 & -r_\mathrm{iy}r_\mathrm{iz} \\ -r_\mathrm{iz}r_\mathrm{ix} & -r_\mathrm{iz}r_\mathrm{iy} & r_\mathrm{ix}^2 + r_\mathrm{i\,y}^2 \end{bmatrix}. \tag{2.2}$$

Here r_ix, $r_{\mathrm{i}y}$, r_iz are the components of the position vector $\boldsymbol{r}_\mathrm{i}^\mathrm{c}$ of the local center of mass of segment „ i “ in the coordinate system $CX_CY_CZ_C$ (fig. 2.2),

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[1] Зациорский, В. М, Аруин, А. С. & Селуянов, В. Н. (1981). Биомеханика двигательного аппарата человека. Физкультура и спорт, Москва. (Zatsiorsky, V. M., Aruin, A. S. & Seluyanov, V. N. (1981). Biomechanics of human motor system. Physical education and Sport, Moscow.)

and $\mathrm{m_i}$ is the mass of the segment. The addend $\mathbf{I}_{\mathrm{tr}}^{\mathrm{c}}$ reflects the shifting of the co-ordinate origin C_i from the local $C_iX_iY_iZ_i$ towards the coordinate system $CX_CY_CZ_C$. The second addend $\mathbf{I}_{\mathrm{rot}}^{\mathrm{c}}$ reflects the rotation of the axes of the local coordinate system $C_iX_iY_iZ_i$ towards the direction of the axes of the coordinate system $CX_CY_CZ_C$, and

$$\mathbf{I}_{\mathrm{rot}}^{\mathrm{c}} = \mathbf{G}_{\mathrm{i}}^{\mathrm{c}}\mathbf{I}_{\mathrm{i}}^{\mathrm{i}}\mathbf{G}_{\mathrm{i}}^{\mathrm{cT}}, \qquad (2.3)$$

where $\mathbf{I}_{\mathrm{i}}^{\mathrm{i}}$ is the tensor of inertia of segment „ i “ in the local coordinate system, $\mathbf{I}_{\mathrm{rot}}^{\mathrm{c}}$ is the tensor of inertia after the rotation (relevant to the coordinate system $CX_CY_CZ_C$), and $\mathbf{G}_{\mathrm{i}}^{\mathrm{c}}$ is a matrix of rotation of the kind (2.4).

2.4.3.3 Determining the orientation of the different body segments. Transformation matrixes

Ususally, one of the segments plays a specific role in presenting the dependencies between the movements. This segment is called basic, and in this model it is the middle part of the torso (B1). In the presented model, the parameters for determining the orientation of the segments are the modified angles of Euler known as rotations x-y-z or angles of Bryan. These angles serve to determine the orientation both of the basic and the other body segments.

The definition of the angles is related to determining the relevant coordinate system (S). We introduce a stationary coordinate system (So) OXYZ, according to which we calculate the global movement. Two coordinate system are attached to each segment. One of them (S_{Ci}) $C_i\ X_i\ Y_i\ Z_i$ is with the origin in the center of gravity of the particular segment C_i (i = 1,...,16). The origin of the other coordinate system (S_{Oi}) $O_i\ x_i\ y_i\ z_i$ is situated at the joint O_i (i = 2,...,16) of the proximal end of the particular segment. These two local coordinate system are fixed to their relevant segment (B_i) and rotate together with it (fig. 2.2). The beginning of the non-rotating coordinate system $CX_CY_CZ_C$ is connected to the center of gravity of the body. The assisting coordinate system $O_0\ x_0\ y_0\ z_0$ with origin in the center of gravity (C_1) of the segment is connected to the basic segment (B_1). This system moves paralel with the system $CX_CY_CZ_C$, i.e., performs only translational movement.

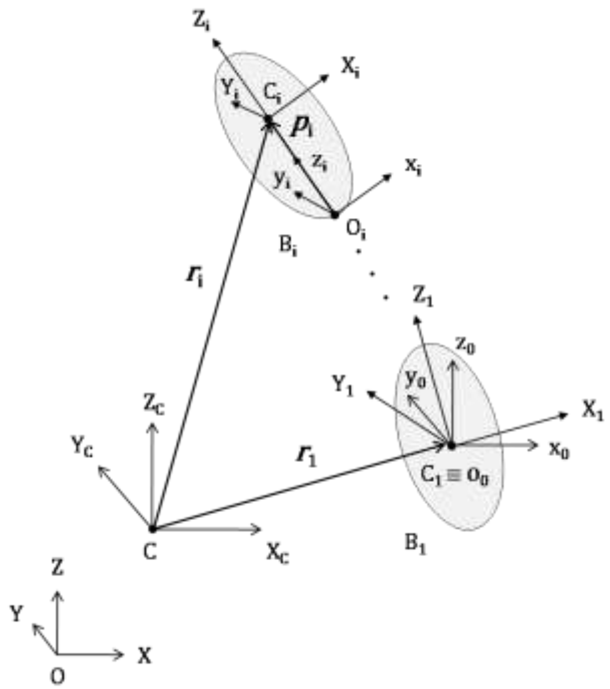

Fig. 2.2 Coordinate system: OXYZ – stationary in space; $CX_CY_CZ_C$ and $O_0\,x_0\,y_0\,z_0$ are non-rotating; $C_iX_iY_iZ_i$ and $O_i\,x_i\,y_i\,z_i$ - fixed at the different segments (B_i)

The applied system of modified angles of Euler (1-2-3) corresponds to the following consecutive rotations:

- rotation of angle Φ around the axis OX, matrix $\mathbf{G}_{x,\phi}$,
- rotation of angle θ around the axis OY', matrix $\mathbf{G}_{y,\theta}$,
- rotation of angle ψ around the axis OZ'', matrix $\mathbf{G}_{z,\psi}$.

The resultant matrix of this transition between the coordinate system is of the kind: $\mathbf{G}_{\phi,\theta,\psi} = \mathbf{G}_{\mathrm{x},\phi}\mathbf{G}_{\mathrm{y},\theta}\mathbf{G}_{\mathrm{z},\psi} =$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \mathrm{c}\phi & -s\phi \\ 0 & \mathrm{s}\phi & \mathrm{c}\phi \end{bmatrix} . \begin{bmatrix} c\theta & 0 & s\theta \\ 0 & 1 & 0 \\ -s\theta & 0 & c\theta \end{bmatrix} . \begin{bmatrix} \mathrm{c}\psi & -s\psi & 0 \\ \mathrm{s}\psi & \mathrm{c}\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} =$$

$$= \begin{bmatrix} c\theta \mathrm{c}\psi & -c\theta s\psi & s\theta \\ c\phi s\psi + s\phi s\theta \mathrm{c}\psi & -s\phi s\theta s\psi + \mathrm{c}\phi \mathrm{c}\psi & -s\phi c\theta \\ s\phi s\psi - c\phi s\theta \mathrm{c}\psi & c\phi s\theta s\psi + s\phi \mathrm{c}\psi & c\phi c\theta \end{bmatrix}, \qquad (2.4)$$

where $\mathrm{s} \equiv \sin$, $\mathrm{c} \equiv \cos$.

We apply this kind of matrix for rotations at the joints with three degrees of freedom. The rotations of the basic segment relativ to the non-rotating coordinate system $O_0\,x_0\,y_0\,z_0$ are also determined with a transition matrix of the kind shown above.

For joints with two degrees of freedom of movement (elbow joints), we use a transition matrix of the kind:

$$\mathbf{G}_{\theta,\psi} = \mathbf{G}_{\mathrm{y},\theta}\mathbf{G}_{\mathrm{z},\psi} = \begin{bmatrix} c\theta c\psi & -c\theta s\psi & s\theta \\ s\psi & c\psi & 0 \\ -s\theta c\psi & s\theta s\psi & c\theta \end{bmatrix}. \qquad (2.5)$$

There are two degrees of freedom at the joints of the wrists and in the movements between the middle and lower part of the torso. The relevant matrix in this transition is of the kind:

$$\mathbf{G}_{\phi,\theta} = \mathbf{G}_{\mathrm{x},\phi}\,\mathbf{G}_{y,\theta} = \begin{bmatrix} c\theta & 0 & s\theta \\ s\phi s\theta & c\phi & -s\phi c\theta \\ -c\phi s\theta & s\phi & c\phi c\theta \end{bmatrix}. \qquad (2.6)$$

One rotation is performed at the knee and ankle joints with transition matrix:

$$\mathbf{G}_{\mathrm{x},\phi} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\phi & -s\phi \\ 0 & s\phi & c\phi \end{bmatrix}. \qquad (2.7)$$

All of the matrixes shown above describe the transition from the coordinate system i to the coordinate system i-1. In order to indicate the direction of the transition, we introduce an upper and a lower index. For instance, the matrix $\mathbf{A}_{\mathrm{i}}^{\mathrm{i-1}}$ is a transition matrix from coordinate system i to coordinate system i-1. For greater briefness of the expressions, we introduce the transition matrixes in rotation and translation as a homogeneous transformation with matrixes of the kind:

$$\mathbf{A}_{\mathrm{i}}^{\mathrm{i-1}} = \mathbf{T}_{\mathrm{i}}^{\mathrm{i-1}}\mathbf{R}_{\mathrm{i}}^{\mathrm{i-1}} =$$

$$= \begin{bmatrix} 1 & 0 & 0 & \\ 0 & 1 & 0 & [\boldsymbol{l}_{\mathrm{i}}^{\mathrm{i-1}}]_{(3\times1)} \\ 0 & 0 & 1 & \\ 0 & 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} & & & 0 \\ & [\mathbf{G}_{\mathrm{i}}^{\mathrm{i-1}}]_{(3\times3)} & & 0 \\ & & & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \mathbf{G}_{\mathrm{i}}^{\mathrm{i-1}} & \boldsymbol{l}_{\mathrm{i}}^{\mathrm{i-1}} \\ \mathbf{0}_{(1\times3)} & 1 \end{bmatrix}, \qquad (2.8)$$

Where the matrix $\mathbf{G}$ is with size 3×3 and is of the kind (2.4), $\boldsymbol{l}_{\mathrm{i}}^{\mathrm{i-1}} = [x_{Oi}^{i-1}\ \ y_{Oi}^{i-1}\ \ z_{Oi}^{i-1}]^{\mathrm{T}}$ is a vector with components the coordinates of the origin O_i of the coordinate system $O_i\,x_i\,y_i\,z_i$, projected in the coordinate system $O_{i-1}\,x_{i-1}\,y_{i-1}\,z_{i-1}$, "T" is a sign for transponation. The matrix $\mathbf{0}_{(1\times3)} = [0 \quad 0 \quad 0]$ is with size 1×3. For instance, if $\boldsymbol{p}_{\mathrm{i}}^{\mathrm{i}} = [x_{Ci} \quad y_{Ci} \quad z_{Ci} \quad 1]^{\mathrm{T}}$ is a position vector of the center of gravity C_i from segment i, presented in the local coordinate system (i.e., it is situated at the proximal end of the segment and connects the origin of $O_i x_i y_i z_i$ and C_i), we can express this vector in the starting coordinate system $CX_CY_CZ_C$ through homogeneous transformations:

$$\boldsymbol{p}_{\mathrm{i}}^{\mathrm{c}} = \mathbf{A}_1^{\mathrm{c}}\mathbf{A}_2^1 \ . \ . \ . \ \mathbf{A}_{\mathrm{i}}^{\mathrm{i}-1}\boldsymbol{p}_{\mathrm{i}}^{\mathrm{i}} \ , \qquad (2.9)$$

where $\mathbf{A}_1^{\mathrm{c}}\mathbf{A}_2^1 \ . \ . \ . \ \mathbf{A}_{\mathrm{i}}^{\mathrm{i}-1}$ are matrixes of transition between the corresponding coordinate systems of the different bodies. Generally, when we have "n" number of bodies, the transition from the local (S_n) to the reference (S_j) coordinate system is made with the matrix:

$$\mathbf{A}_{\mathrm{n}}^{\mathrm{j}} = \prod_{\mathrm{i=j}}^{\mathrm{n}-1} \mathbf{A}_{\mathrm{i}+1}^{\mathrm{i}}$$

2.4.3.4 Equations of motion

When we work out the equations of the body motion, we apply the fundamental principle known as Law for conservation the size and direction of the vector of angular momentum in relativ to non-rotating coordinate system with origin the center of gravity of the body. In vector form, the expression of the angular momentum is of the kind:

$$\boldsymbol{H}_{\mathrm{c}} = \sum_{\mathrm{i}=1}^{\mathrm{n}}(\boldsymbol{r}_{\mathrm{i}}^{\mathrm{c}} \times \mathrm{m}_{\mathrm{i}}\dot{\boldsymbol{r}}_{\mathrm{i}}^{\mathrm{c}} + \ [\mathbf{I}_{\mathrm{i}}\boldsymbol{\Omega}_{\mathrm{i}}]^{\mathrm{c}}), \qquad (2.10)$$

where: $\boldsymbol{H}_{\mathrm{c}}$ is the angular momentum of the body in relation to the center of gravity; m_{i} is the mass of the segment i; $\boldsymbol{r}_{\mathrm{i}}^{\mathrm{c}}$ is the radius-vector of the center of gravity of segment i (and the corresponding derivative $\dot{\boldsymbol{r}}_{\mathrm{i}}^{\mathrm{c}}$) in the non-rotating coordinate system $CX_CY_CZ_C$; $\mathbf{I}_{\mathrm{i}}$ is the tensor of inertia of segment i; $\boldsymbol{\Omega}_{\mathrm{i}}$ is the angular velocity of segment i. The product $\mathbf{I}_{\mathrm{i}}\boldsymbol{\Omega}_{\mathrm{i}}$ (local angular momentum) is also presented in the references coordinate system (S_C). We determine the vectors $\boldsymbol{r}_{\mathrm{i}}^{\mathrm{c}}$, by applying the model of the closed contours. For instance, we present the vector $\boldsymbol{r}_{\mathrm{i}}^{\mathrm{c}}$ (figr.2.2) in the references coordinate system $CX_CY_CZ_C$ with the expression:

$$\boldsymbol{r}_{\mathrm{i}}^{\mathrm{c}} = \boldsymbol{r}_1^{\mathrm{c}} + \mathbf{R}_1^0\mathbf{A}_2^1 \ ... \ \mathbf{A}_{\mathrm{i}}^{\mathrm{i}-1}\boldsymbol{p}_{\mathrm{i}}^{\mathrm{i}} \qquad (2.11)$$

For greater convenience, instead of a matrix of transition $\mathbf{A}_1^{\mathrm{c}}$, at the beginning of the right part of the equations of the kind (2.11) we use the expression $\boldsymbol{r}_1^{\mathrm{c}} + \mathbf{R}_1^0$. This will facilitate us later on when we work out the unknown derivatives $\dot{\phi}_1$, $\dot{\theta}_1$ и $\dot{\psi}_1$.

When we work out the equations of motion, we use the expression for the center of mass of a system of bodies:

$$m\boldsymbol{r}_c = \sum_{i=1}^{n} m_i \boldsymbol{r}_i \;, \quad m = \sum_{i=1}^{n} m_i \qquad (2.12)$$

Provided the coordinate system's origin is the center of mass, $\boldsymbol{r}_c = 0$, and:

$$0 = m_1 \boldsymbol{r}_1^c + \ldots + \sum_{i=2}^{16} m_i \boldsymbol{r}_i^c$$

and after some transformations, we work out the expression for $\boldsymbol{r}_1^c$.

We need to differentiate the matrixes $\mathbf{R}_1^0$ and $\mathbf{A}_i^{i-1}$ (i = 2,...,16) of the kind (2.11), so we can find the velocities of $\dot{\boldsymbol{r}}_i^c$ of the center of gravity of the different segments. We know from theoretical mechanics (Angelov, 2005, 2008)[2] that it can be done in the following way:

$$\dot{\mathbf{R}}_1^0 = \widetilde{\mathbf{W}}_1^0 \mathbf{R}_1^0 \qquad (2.13)$$

and from (2.8) $\dot{\mathbf{A}}_i^{i-1} = \dot{\mathbf{T}}_i^{i-1}\mathbf{R}_i^{i-1} + \mathbf{T}_i^{i-1}\dot{\mathbf{R}}_i^{i-1} = \mathbf{T}_i^{i-1}\widetilde{\mathbf{W}}_i^{i-1}\mathbf{R}_i^{i-1} = \widetilde{\mathbf{W}}_i^{i-1}\mathbf{R}_i^{i-1}$ (2.14)

after taking into account that $\dot{\mathbf{T}}_i^{i-1} = 0$ ($\boldsymbol{l}$ = const at i = 2,...,16). Next, we have to explicate the matrix $\widetilde{\mathbf{W}}_1^0$**.** The sign "~" denotes an antisimmetrical kind of presentation, i.e.:

$$\widetilde{\mathbf{W}}_1^0 = \begin{bmatrix} 0 & -\boldsymbol{\Omega}_{z1}^0 & \boldsymbol{\Omega}_{y1}^0 & 0 \\ \boldsymbol{\Omega}_{z1}^0 & 0 & -\boldsymbol{\Omega}_{x1}^0 & 0 \\ -\boldsymbol{\Omega}_{y1}^0 & \boldsymbol{\Omega}_{x1}^0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} \widetilde{\boldsymbol{\Omega}}_1^0 & \mathbf{0}^T \\ \mathbf{0} & 0 \end{bmatrix}, \qquad (2.15)$$

The vectors $\mathbf{W}_1^0 = \begin{bmatrix} \boldsymbol{\Omega}_{x1}^0 & \boldsymbol{\Omega}_{y1}^0 & \boldsymbol{\Omega}_{z1}^0 & 0 \end{bmatrix}^T$ and $\boldsymbol{\Omega}_1^0 = \begin{bmatrix} \boldsymbol{\Omega}_{x1}^0 & \boldsymbol{\Omega}_{y1}^0 & \boldsymbol{\Omega}_{z1}^0 \end{bmatrix}^T$ are the same vector but with different size – respectively (4×1) and (3×1). This is the vector of the angular velocity of segment 1, projected in the references coordinate system $O_0\, x_0\, y_0\, z_0$. We continue supplying the needed operators by sticking to Angelov's explanations (2005, 2008). The transition to the references coordinate system $O_0\, x_0\, y_0\, z_0$, for the angular velocity of segment 1, expressed with angles of Bryan $\dot{\boldsymbol{\Theta}}_1 = \begin{bmatrix} \dot{\phi}_1 & \dot{\theta}_1 & \dot{\psi}_1 \end{bmatrix}^T$, is made with the help of the matrix:

[2] Ангелов, Ил. В. (2005). Матрична механика. Авангард Прима, София.ISBN 954-323-110-9 (Angelov, Il. V. (2005). Matrix Mechanics. Avant Gard Prima, Sofia)

Ангелов, Ил. В. (2008). Матрична механика кинематика. Авангард Прима, София. ISBN 978-954-323-417-2 (Angelov, Il. V. (2008). Matrix mechanics Kinematics. Avant Ggard Prima, Sofia)

$$\mathbf{U}_1^{\Omega 0} = \begin{bmatrix} 1 & 0 & \mathrm{s}\theta_1 \\ 0 & \mathrm{c}\phi_1 & -\mathrm{s}\phi_1 \mathrm{c}\theta_1 \\ 0 & \mathrm{s}\phi_1 & \mathrm{c}\phi_1 \mathrm{c}\theta_1 \end{bmatrix}. \tag{2.16}$$

Transition matrixes of the kind $\mathbf{U}_1^{\Omega 0}$ are applied with the segments with three rotation degrees of freedom. After determining the other similar transition matrixes for the segments with two and one degree of freedom and the corresponding angular velocities, we have all the matrixes of the kind $\widetilde{\mathbf{W}}_{\mathrm{i}}^{\mathrm{i}-1}$, needed for differentiation of the corresponding matrixes $\dot{\mathbf{A}}_{\mathrm{i}}^{\mathrm{i}-1}$. After numerous substitutions, groupings, and transformations, in the end we work out the components of the first addend of the expression (2.10) of the angular momentum $\boldsymbol{H}_{\mathrm{c}}$, often called orbital angular momentum. Now we have to work out the second addend of the expression (2.10), known as a local angular momentum. Here, we work with vectors and matrixes with sizes (3×1) and (3×3). The local angular momentum $\mathbf{H}_{c,local}$ is expressed in relation to the references coordinate system $O_0\, x_0\, y_0\, z_0$, which moves parallel with the system $CX_CY_CZ_C$ and performs only one translational movement. In some cases, however, it is more rational to project vectors on the axes of the local coordinate system in intermediate operations. We do so in this case. First, we find $\mathbf{H}_{\mathrm{i},local}$ in relation to its own (local) coordinate system where the tensor of inertia of each segment is of diagonal type, and its components are known:

$$\mathbf{I}_{\mathrm{i}}^{\mathrm{i}} = \begin{bmatrix} I_{\mathrm{xxi}} & 0 & 0 \\ 0 & I_{\mathrm{yyi}} & 0 \\ 0 & 0 & I_{\mathrm{zzi}} \end{bmatrix}, \tag{2.17}$$

where I_{xxi}, I_{yyi}, I_{zzi} are the components of the tensor of inertia in relation to the main axes of segment i. We continue by preparing another set of matrixes, serving when projecting the angular velocity between segment i and segment i-1 $\dot{\mathbf{\Theta}}_{\mathrm{i}} = \begin{bmatrix} \dot{\phi}_{\mathrm{i}} & \dot{\theta}_{\mathrm{i}} & \dot{\psi}_{\mathrm{i}} \end{bmatrix}^{\mathrm{T}}$ in the own coordinate system of segment i - $C_i\, X_i\, Y_i\, Z_i$, which is presented by $\boldsymbol{\Omega}_{\mathrm{i}}^{\mathrm{i}}$. In this case, the transition matrix is of the kind:

$$\mathbf{U}_{\mathrm{i}}^{\Omega \mathrm{i}} = \begin{bmatrix} \mathrm{c}\theta_{\mathrm{i}} \mathrm{c}\psi_{\mathrm{i}} & \mathrm{s}\psi_{\mathrm{i}} & 0 \\ -\mathrm{c}\theta_{\mathrm{i}} \mathrm{s}\psi_{\mathrm{i}} & \mathrm{c}\psi_{\mathrm{i}} & 0 \\ \mathrm{s}\theta_{\mathrm{i}} & 0 & 1 \end{bmatrix}, \tag{2.18}$$

Transition matrixes of the kind $\mathbf{U}_{\mathrm{i}}^{\Omega \mathrm{i}}$ are applied for the segments with three degrees of freedom in the corresponding rotations. Then, we determine the transition matrixes of the angular velocities in their own coordinate systems and those of the segments with two or one degree of freedom, as well as the corresponding angular velocities. We know from theoretical mechanics that the vector of the absolute angular velocity ($\boldsymbol{\Omega}_{\mathrm{i}}^{\mathrm{i}}$) of a particular segment i from the system of connected segments (n), projected on the axes of the local coordinate system $C_i X_i Y_i Z_i$, is the sum of the relative angular velocities (of i in relation to i-1) of the bodies ($\omega_{\mathrm{i-1,i}}$, i = 1...n), projected in the local coordinate system $C_i X_i Y_i Z_i$:

$$\boldsymbol{\Omega}_{\mathrm{i}}^{\mathrm{i}} = \omega_{0,1}^{\mathrm{i}} + \omega_{1,2}^{\mathrm{i}} + \ ... + \omega_{\mathrm{i-1,i}}^{\mathrm{i}} \qquad (2.19)$$

In this case, at a relative angular velocity, presented with angles of Bryant, ($\dot{\boldsymbol{\Theta}}_{\mathrm{i}}$) the expression (2.19) takes the following kind:

$$\boldsymbol{\Omega}_{\mathrm{i}}^{\mathrm{i}} = \mathbf{U}_{1}^{\Omega 1}\dot{\boldsymbol{\Theta}}_{1}, \text{ at i=1;}$$

$$\boldsymbol{\Omega}_{\mathrm{i}}^{\mathrm{i}} = \mathbf{G}_{\mathrm{i}}^{1\mathrm{T}}\mathbf{U}_{1}^{\Omega 1}\dot{\boldsymbol{\Theta}}_{1} + \mathbf{U}_{\mathrm{i}}^{\Omega \mathrm{i}}\dot{\boldsymbol{\Theta}}_{\mathrm{i}} \text{ at i=2;}$$

$$\boldsymbol{\Omega}_{\mathrm{i}}^{\mathrm{i}} = \mathbf{G}_{\mathrm{i}}^{1\mathrm{T}}\mathbf{U}_{1}^{\Omega 1}\dot{\boldsymbol{\Theta}}_{1} + \ ... + \mathbf{G}_{\mathrm{i}}^{\mathrm{i-1T}}\mathbf{U}_{\mathrm{i-1}}^{\Omega \mathrm{i-1}}\dot{\boldsymbol{\Theta}}_{\mathrm{i-1}} + \mathbf{U}_{\mathrm{i}}^{\Omega \mathrm{i}}\dot{\boldsymbol{\Theta}}_{\mathrm{i}} \text{ at i > 2,} \qquad (2.20)$$

Taking into consideration that $\mathbf{G}_{\mathrm{i}}^{1\mathrm{T}} = \mathbf{G}_{\mathrm{i}}^{\mathrm{i-1T}} \ ... \ \mathbf{G}_{2}^{1\mathrm{T}}$

After expressing the product $\mathbf{I}_{\mathrm{i}}^{\mathrm{i}}\boldsymbol{\Omega}_{\mathrm{i}}^{\mathrm{i}}$ (local angular momentum) in the coordinate system of segment i, we can easily, with the help of the resultant matrix $\mathbf{G}_{\mathrm{i}}^{0}$ of transition between the coordinate systems, present the components of the product $\mathbf{I}_{\mathrm{i}}^{\mathrm{i}}\boldsymbol{\Omega}_{\mathrm{i}}^{\mathrm{i}}$, projected in the references coordinate system $O_0 x_0 y_0 z_0$ (S_0):

$$\mathbf{H}_{\mathrm{i},local}^{\mathrm{i}} = \mathbf{I}_{\mathrm{i}}^{\mathrm{i}}\boldsymbol{\Omega}_{\mathrm{i}}^{\mathrm{i}},$$

$$\mathbf{H}_{\mathrm{i},local} = \mathbf{G}_{\mathrm{i}}^{0}\mathbf{H}_{\mathrm{i}\,\mathrm{i},local}^{\mathrm{i}} \qquad (2.21)$$

The further preparation of the equations for computer realization includes numerous substitutes and groupings, and in the final phase, we obtain a system of three ordinary differential equations (ODE) of first order, and after solving them, we can obtain the orientation of the basic segment 1 (and of all the other segments in the same way), i.e., the angles ϕ_1, θ_1 and ψ_1 during the movement.

2.4.3.5 Control of the model

In order to solve the differential equations of the motion we have to know the values of the angles $\boldsymbol{\Theta}_{\mathrm{i}}$ and the angular velocity $\dot{\boldsymbol{\Theta}}_{\mathrm{i}}$ in the joints during the movement (i = 2,...,16). Assigning the relative movements in the joints, in fact, we control the behaviour of the model.

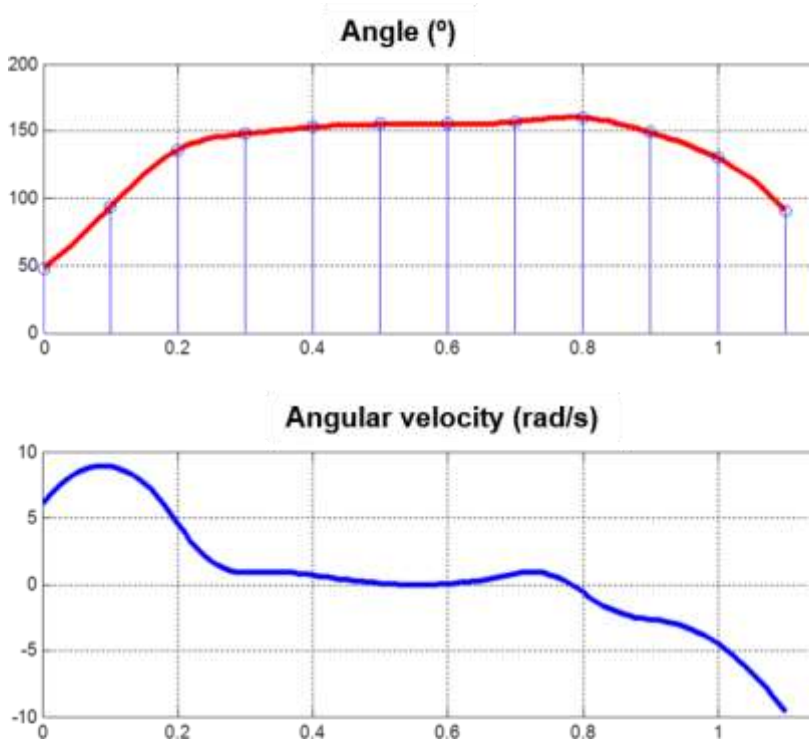

Fig. 2.3 Interpolation of the change in the angle in the shoulder joint (flexion/extension) with cubic spline function (with 12 knots)

We use two approaches in the control of the model. In the first approach, first we built the graphs of the functions of the change in the angles over time. Building the curves angle-time is made with cubic spline function (fig. 2.3). We view the obtained form of the curve and then we build a differential curve angular velocity-time.

In the other approach, in order to gain control over the relative movements in joint angles, the time for a particular movement is divided into equal in duration subintervals (usually the duration of the subintervals ranges from 0.04 to 0.1 sec). It is accepted that the angular velocity of each subinterval is constant and equal to the average angular velocity for the change in the angle in this subinterval. The vectors with the values of angular velocities for the different subintervals are created.

2.4.3.6 Solving the equations of the motion. Program for numerical experiments

In order to solve the differential equations of the motion, we have to input the following initial data: antropometric and mass-inertia parameters of the different segments; the values of the angles describing the initial orientation of the basic segment; the values of the angles in the joints at the starting moment (initial body configuration) and during the performance, as well as the angular velocity of their change; the values of the components of the angular momentum of the body in relation to the center of gravity. After preparing the equations for computer realization, we have to select the method for solving them. In this case we applied the function (solver 113) of the program system MATLAB for numerical integration of a system of nonlinear ODE based on the method of Adams-Bashforth-Moulton with variable order (Yordanov, 2004)[3].

The simulations of the researched movements are realized with a program which includes several program modules with certain operational functions. The program was developed and functions in the computing setting MATLAB. The created author's program for conducting numerical experiements and calculations consists of 45 program files (M-file), of which: main program – 17 files; biomechanical calculations – 6 files; initial conditions – 11 files; gymnastic elements and movements – 11 files. There is a program archive of vectors of control of: 51 elements; 115 trials of the elements; 47 experimental movements.

The solution, i.e., the output arguments, is a vector column with the values of time (t) for the different steps in the integration, and a matrix whose elements are the values of the three angles describing the orientation of the basic segment for the particular element of the vector of time (t).

2.4.3.7 Visualization of the results. Anthropomorphic structure

After the numerical solution of the differential equation, we obtain the values of the three angle coordinates (ϕ_1, θ_1 and ψ_1) of the basic segment for each step in the integration process. These values can be presented as an array of numbers in a table format or in a graphic format – curves as functions of time. Their presentation in this way, however, impedes significantly the perception of the simulated movement. We think that the rational solution is the design of an author's program for visualization of the current results (fig. 2.4). In this way, we can obtain a clear vision of the movement during the very process of integration.

[3] Йорданов, Й. Т. (2004). Приложение на MATLAB в инженерните изследвания. Част II. Русенски университет, Русе, ISBN 954-712-232-0 (Yordanov, Y. T. (2004). Application of MATLAB in engineering research. Part II. Rouse University, Rouse)

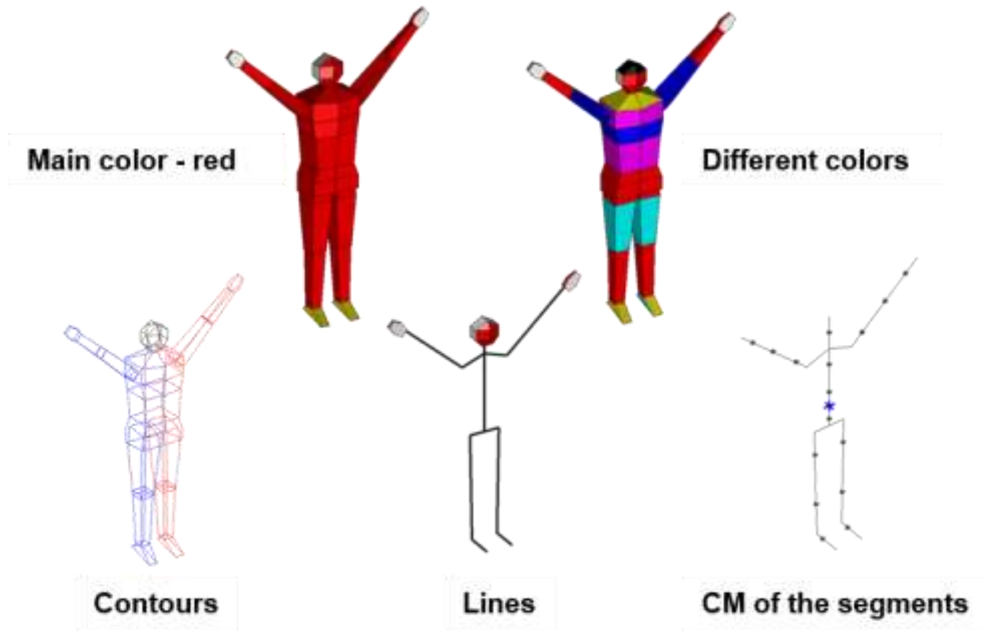

Fig. 2.4 Options for body visualization - (3D) anthropomorphic structures

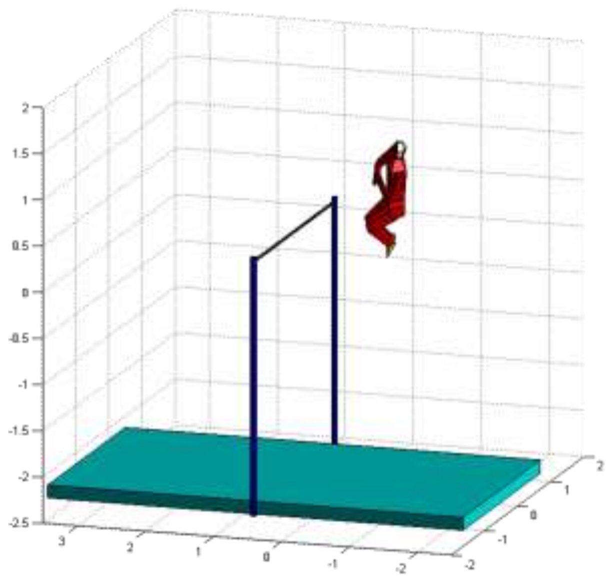

Fig. 2.5 Sample point of view when observing a simulated movement

The realization of the assigned movement can be observed from different points of view by positioning “the camera” according to the specificity of the movements. We have to set the azimuth angle (horizontal rotation around the vertical axis) and the slope of the horizontal plane (fig. 2.5).

Our full attention is directed to the rotation components of the movement. That’s why during the observations we can neglect the translational parabolic movement, determined by three simple equations describing the shift in the center of gravity. The selection of the point of view is determined by the type of the movement (fig. 2.6).

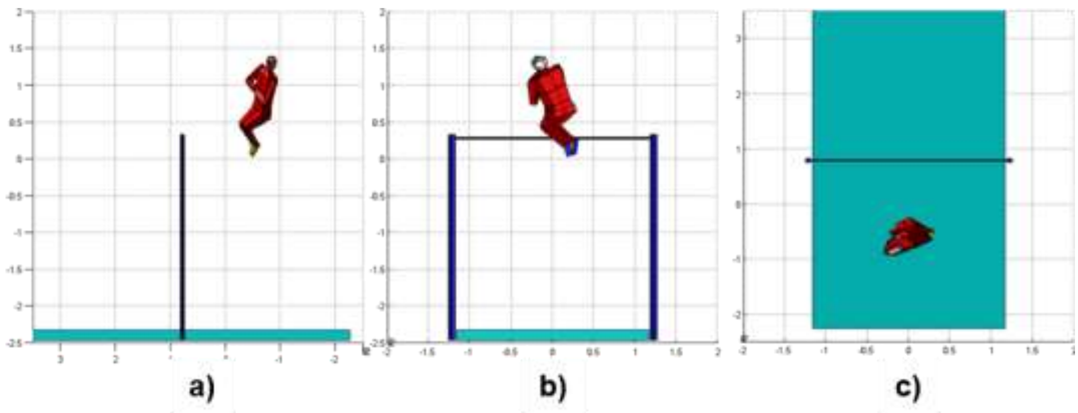

Fig. 2.6 Options of points of view depending on the observed movement: (a) from the side; (б) in front; (в) from the top

2.4.3.8 Evaluation of the model

Before the beginning of the numerical experiemnts, we check the immitation qualities of the model. The validation is made by comparing the data received from the practical execution of certain movements with the data obtained from the simulations with the model of the same movements under the same mechanical conditions (fig. 2.7).

Fig. 2.7 Comparison between the practical execution (above) and simulation (below) of a dismount off horizontal bars – double back semi-tuck somersault with a twist - 360°

In this case, we made a quality evaluation of the suitability of the model to simulate a real execution taken from gymnastics practice. We used a digital recording camera (DSLR Camera – Nikon D700 with battery grip MB-D10 and Nikkor 24-70 mm lens). Finally, we can conclude that in this case an entirely satisfactory similarity between the images from simulation and video recording is achieved. An additional validation was also made by comparing the results about the angle of variations of the tilt between theory and simulation. There is a full conformity between the angles obtained analitically and from the simulation with the model. At a starting angle of the tilt $\theta_0 = 3.7042$ after 90° turn $\theta_1(\text{theory}) = 5.10172$, and $\theta_1(\text{simulation}) = 5.101698$.

2.4.3.9 Determining the athlete's body orientation through the principal axes of inertia

We know from theoretical mechanics that we can easily determine the direction of the principal axes of inertia if we know the direction of one of them (e.g., Greenwood, 1988)[4].

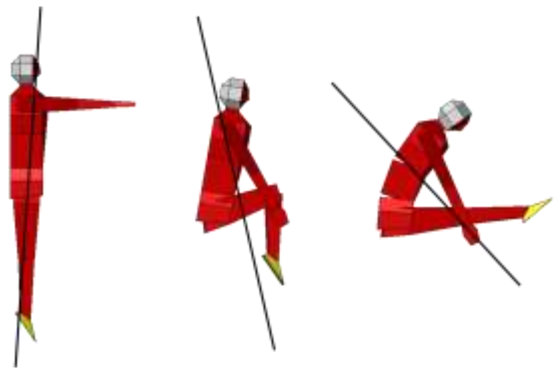

Fig. 2.8 Direction of the principal axis of inertia (rotation of angle ϕ) in different body configurations

In this case, the rotation is around the principal axis of inertia CX, therefore $I_{xy}(I_{yx}) = I_{xz}(I_{zx}) = 0$. To establish the direction of the longitudinal principal axis of inertia, we have to find the angle of its rotation (ϕ) around axis CX. In order to calculate the value of the angle ϕ, we apply a formula from theoretical mechanics:

$$\text{tg}2\phi = 2I_{yz}/(I_{yy} - I_{zz}). \tag{2.23}$$

We apply an identical approach to find the angle of the tilt of the body, i.e., the inclination sideways from sagittal plane.

[4] Greenwood D. T. (1988). Principles of Dynamics, 2nd ed., Prentice-Hall, Englewood Cliffs, NJ.

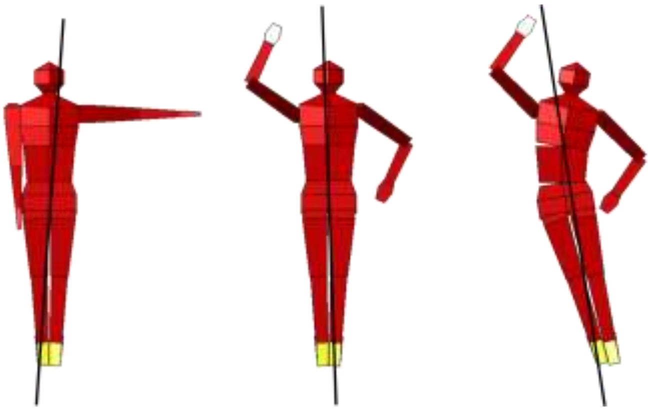

Fig. 2.9 Direction (rotation of angle θ) of the principal longitudinal axis of inertia in different body configurations

2.4.4 Numerical experiments

The numerical experiments were conducted in two directions. On the one hand, we conducted experiments for enriching and specifying some theoretical knowledge, as well as for confirming established facts. On the other hand, we conducted numerous experiments for establishing a certain desired behaviour and finding a motor optimality in accordance with the aim of the research.

2.4.5 Biomechanical analysis

Along with the 16-segment model, we developed some subprograms for calculation of biomechanical quantities such as: 3D coordinates of the center of gravity of the body and the local centers of gravity of the segments; the components of the velocity of the center of gravity of the body and the local centers of gravity of the segments; the components of angular velocity and angular momentum; the components of the tensors of inertia of all segments.

To make a comparison between the mechanical quantities, calculated for athletes with different mass-inertia and anthropometric data, in some case, we perform the procedure **normalization** of certain quantity. In order to receive the equivalent number of straight somersaults during the flight time, Hiley and Yeadon (2008)[5] clarified the term rotation potential which is calculated as a product of the angular momentum and flight time, divided by 2π times the inertia moment when the body is straight We use this approach and the term rotation potential in our survey.

[5] Hiley, M. J. & Yeadon, M. R. (2008). Optimisation of high bar circling technique for consistent performance of a triple piked somersault dismount. *Journal of Biomechanics*, 41 (8), 1730–1735.

3 Results and analysis

3.1 Rotational motions of the body during the non-support phase

Our interest is fully focused on the rotary motion around the center of mass of the body. The combination of somersaults and twists is viewed as rotation around a stationary point (the center of mass). This kind of movement is known as the case Euler-Poinsot. The rotation is made around a instantaneous axis of rotation which describes cone planes in the course of the movement.

3.1.1 Creation of rotational motion of the body around the longitudinal axis during the non-support phase

It is well known that during the non-support phase the rotational motion around the longitudinal axis can be created in two ways. The first requires a rotational motion around the axis of somersault during the non-support phase. The second way not requires the presence of any rotational motion, and here the mechanisms of counter rotation are mainly applied ("hula hoop" technique).

3.1.1.1 Creation of rotational motion of the body around the longitudinal axis during the non-support phase in the presence of rotational motion around the lateral axis (the axis of the somersault)

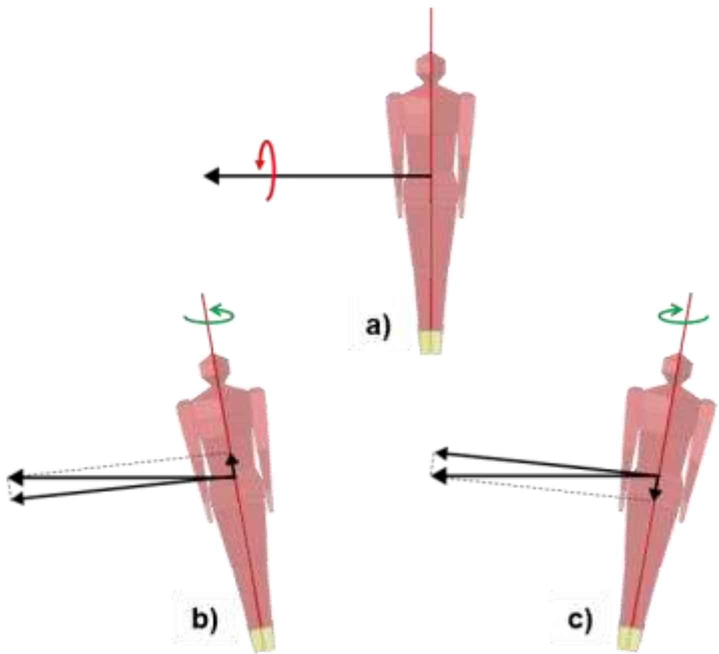

Fig. 3.1 Angular momentum in straight back somersault (a). Creation of rotational motion around the longitudinal axis with a tilt: tilt to the right – rotation to the left (b); tilt to the left – rotation to the right (c)

In order that a rotational motion around the longitudinal axis be created, there should be a component of the angular momentum on the direction of this axis (fig. 3.1b and fig. 3.1c). This can be achieved if the longitudinal principal axis of inertia of the body is inclined in relation to the direction of the angular momentum. In this case, there is a projection of the angular momentum on the direction of the longitudinal principal axis of inertia, i.e., part of the rotational motion of the somersault is "redirected" for the twist of the body as well.

With the increase of the magnitude of the lateral inclination, the quantity of the twists is also increased. After simulations of execution of straight back somersault with one, two or three twists (configuration – straight body, arms down to the sides, fly time – 1 sec), we obtained the results shown in fig. 3.6.

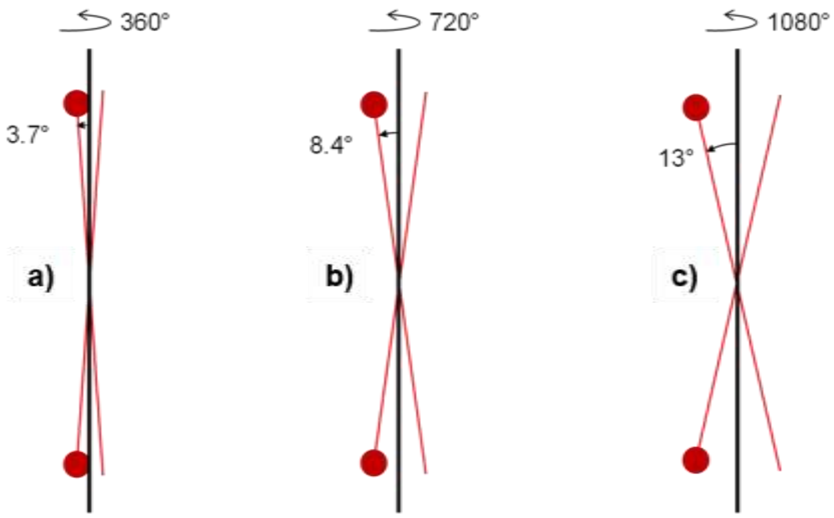

Fig. 3.6 Angle of the tilt in straight back somersault depending on the quantity of the twists: (a) at 360°; (b) at 720°; (c) at 1080°

3.1.1.2 Body tilt without a rotational motion for the somersault

During the certain actions, the angular momentum remains zero, which means that if part of the segments rotate in one direction, the other body segments will rotate in the opposite direction. The quantity of the rotations depends on the anthropometric and mass-inertia characteristics of the segments.

Fig. 3.9 shows the effect on the magnitude of the tilt when lowering one arm down to the front from initial position with arms up. The final magnitude of the tilt is a result from the value of the tilt after lowering the arm, plus a little value from the additional inclination of the principal axis of inertia in the assimetric final position of the arms.

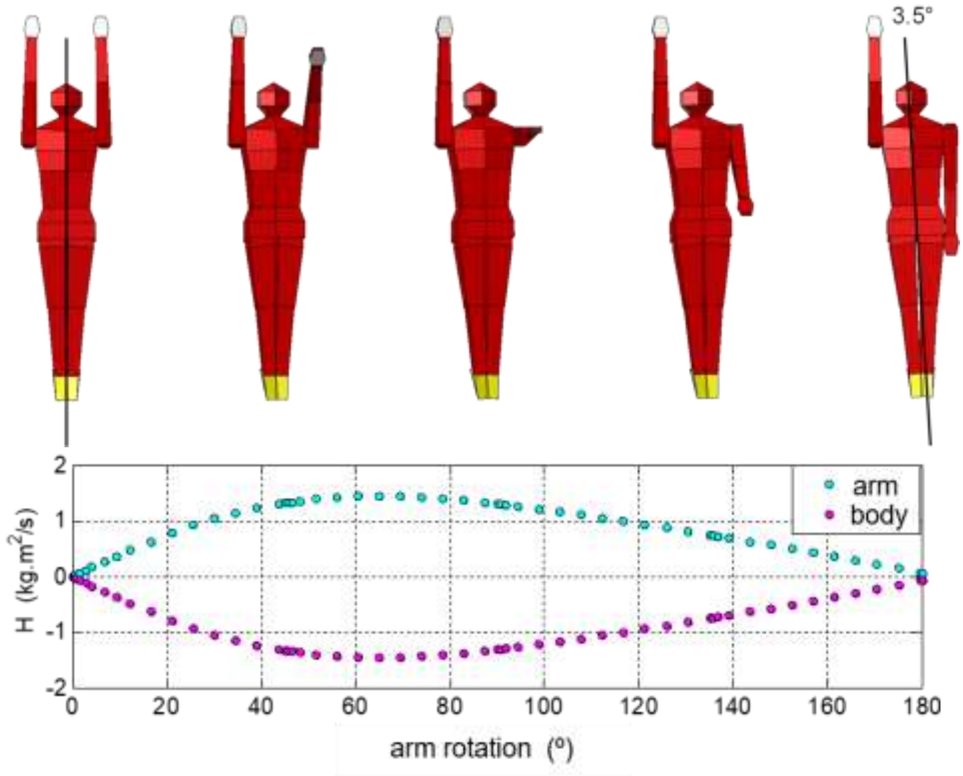

Fig. 3.9 Angular momentum (H_y) of the left arm (●) and the other body segments (●) when lowering the arm to the front

We establish the influence of another typical action – lowering of one arm down sidewards on the magnitude of the tilt (fig. 3.10 above). In accordance to what we expected, the inclination of the principal longitudinal axis of inertia is bigger than in the previous example - 7.3° and is the result from the movement of the arm (5.77°) plus the additional inclination of the principal longitudinal axis of inertia (1.55°) as a result of the assymetrical final position of the arms.

Due to the small inertia momentum of the arm ($\mathbf{I}_p$) the magnitude of the value of the angular momentum of the arm is mainly based on the movement of the local CM of the arm. This peculiarity is shown in the graph in figure 3.10. Obviously, in order to achieve a bigger body tilt when lowering the arm, the partial center of mass of the arm should move in a bigger distance from the sagittal plane. This can be achieved through lowering the straight arm to the side.

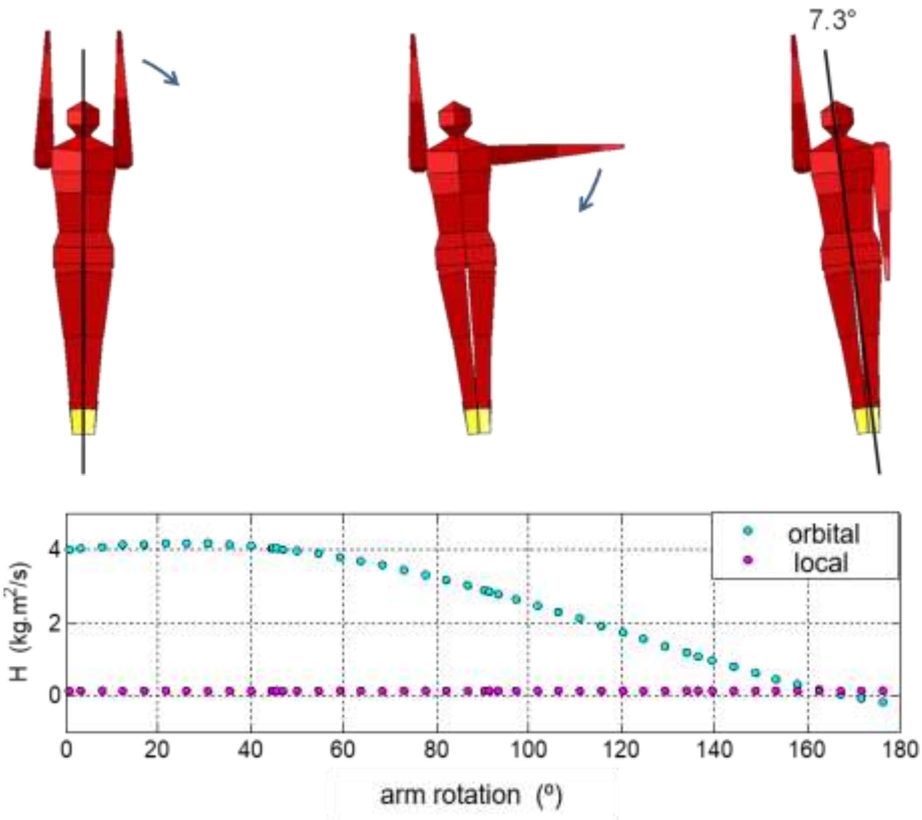

Fig. 3.10 Components of the angular momentum of the left arm:
$\boldsymbol{r}_{\mathrm{p}}^{\mathrm{c}} \times \mathrm{m}_{\mathrm{p}}.\dot{\boldsymbol{r}}_{\mathrm{p}}^{\mathrm{c}}$ ( ) and $\mathbf{I}_{\mathrm{p}}\boldsymbol{\Omega}_{\mathrm{p}}$ ( ) when lowering the arm to the side

We can further increase the magnitude of the tilt of the principal longitudinal axis of inertia (7.9°) if, from initial position with arms to the sides, one arm is raised up and the other arm is lowered down to the side (fig. 3.11).

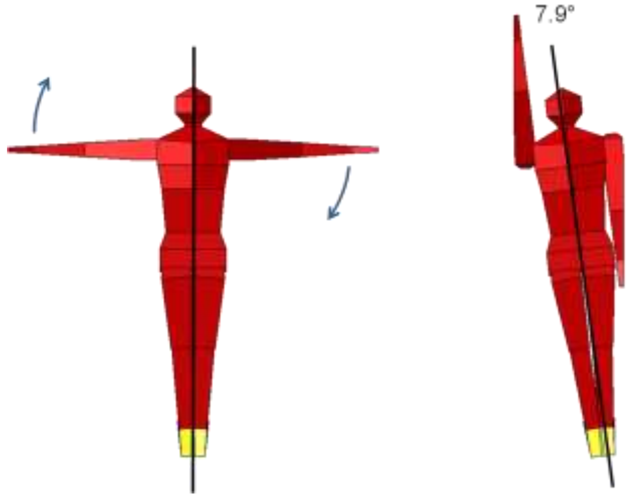

Fig. 3.11 Body tilt when raising one arm up and lowering the other arm down from initial position arms to the sides

We can see in figure 3.12 that when we have a lateral flexion of the torso in one direction, the principal longitudical axis of inertia of the body inclines in the opposite direction. In position with arms down and a lateral flexion of the torso about 12° (6° for each of the two joint connections), the sideward inclination of the principal longitudinal axis of inertia is minimal - 0.5°. This inclination can be slightly increased, to about 2.6° if, when performing the lateral flexion of the torso the arms are to the sides. The bigger lateral flexion of the torso leads to a bigger lateral inclination of the principal longitudinal axis of inertia, but on the one hand, the possibilities for increasing the magnitude of the lateral flexion of the torso are greatly limited, and on the other hand, this could lead to significant technical difficulties in the execution.

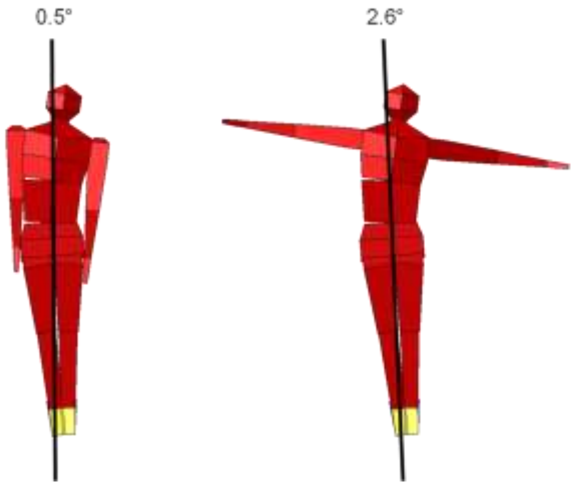

Fig. 3.12 Lateral inclination of the principal longitudinal axis of inertia of the body when performing a lateral flexion of the torso

In the above mentioned movements with the arms we can slightly increase the magnitude of the lateral inclination of the principal longitudinal axis of inertia if the particular movement is executed together with a lateral flexion of the torso.

3.1.1.3 Body tilt with a rotational motion for the somersault

First, we examine the peculiarities and effective actions for creating a maximum quantity of rotational motion around the longitudinal axis in the presence of a great quantity of rotational motion around the axis of the somersault (e.g., two straight somersault). After that we will adjust (simplify) the established actions for the exercises which require less quantity of rotary motion around these axes. Starting learning the exercises with combinations of rotations, gymnasts will have a clear idea what actions they should take in the simplest elements so that afterwards, they can easily (due to the motor similarity) modify these actions into highly efficient motor actions needed for the execution of more complex gymnastics elements with a greater quantity of twists.

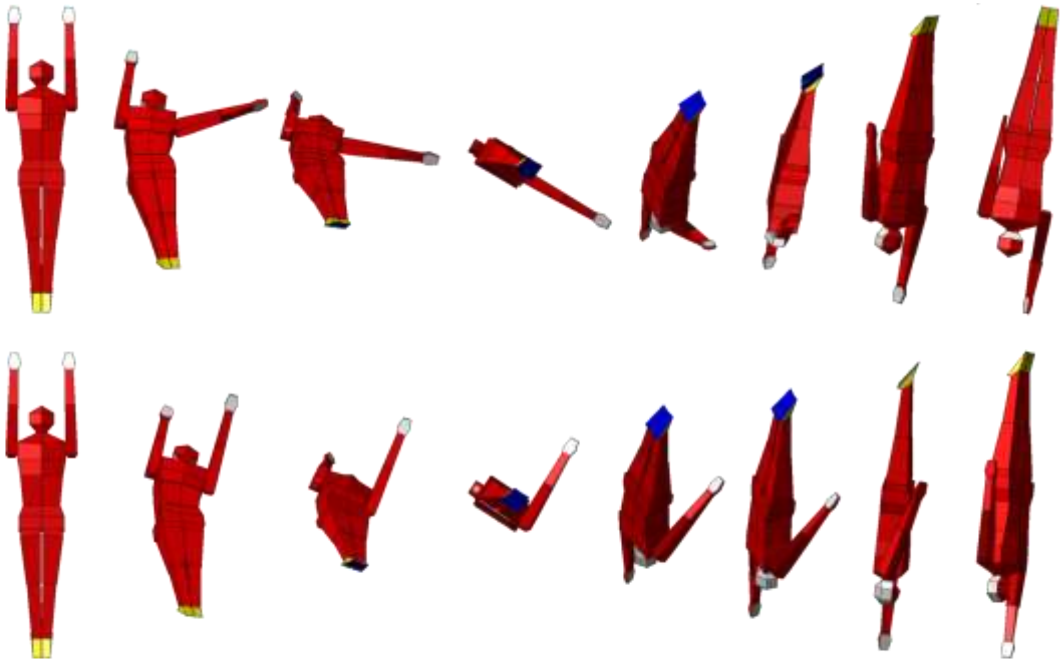

Fig. 3.13 Lowering one arm to the side (up) and to the front (down) with rotational motion for back somersault

In general, lowering the arm to the side, as a whole, leads to a greater lateral inclination and greater twist of the body than lowering the arm to the front. When performing the same motor actions, shown in fig. 3.13, but provided that the direction of rotation is forward, we can achieve a significantly smaller twist.

3.1.2 Motor actions for increasing the quantity of the twists

3.1.2.1 In straight somersault

After performing a number of simulations initiating a twist, we found that when perfoming the actions shown in figure 3.17, we can achieve the greatest quantity of the twists.

If we flexing the arms, the effect on the lateral inclination of the body is smaller.

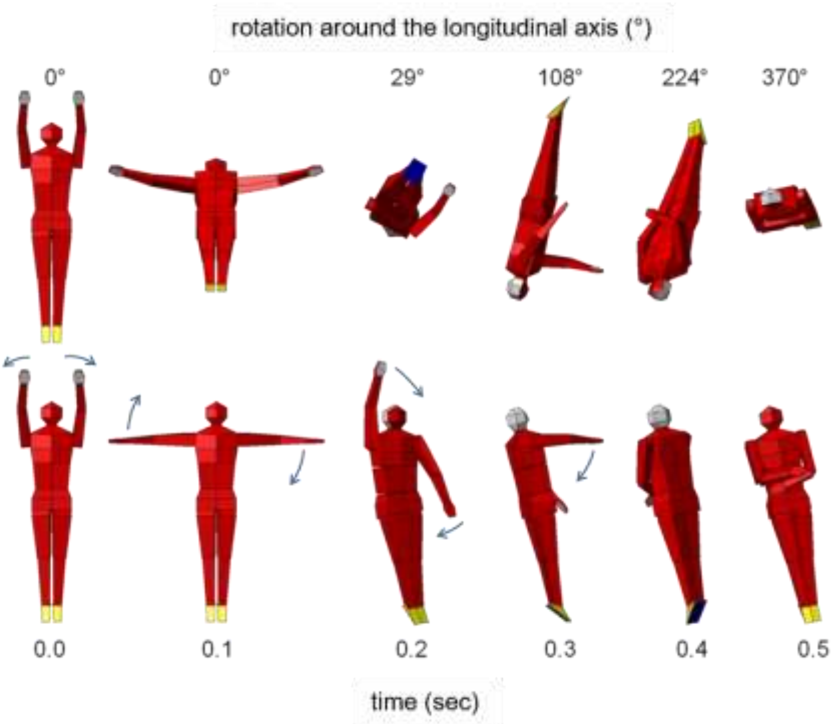

Fig. 3.17 Increasing the quantity of the twists through lateral flexion of the torso (view – to the front). In the sequence of images below, the rotation movement of the somersault is not presented

3.1.2.2 In transition from pike to straight somersault

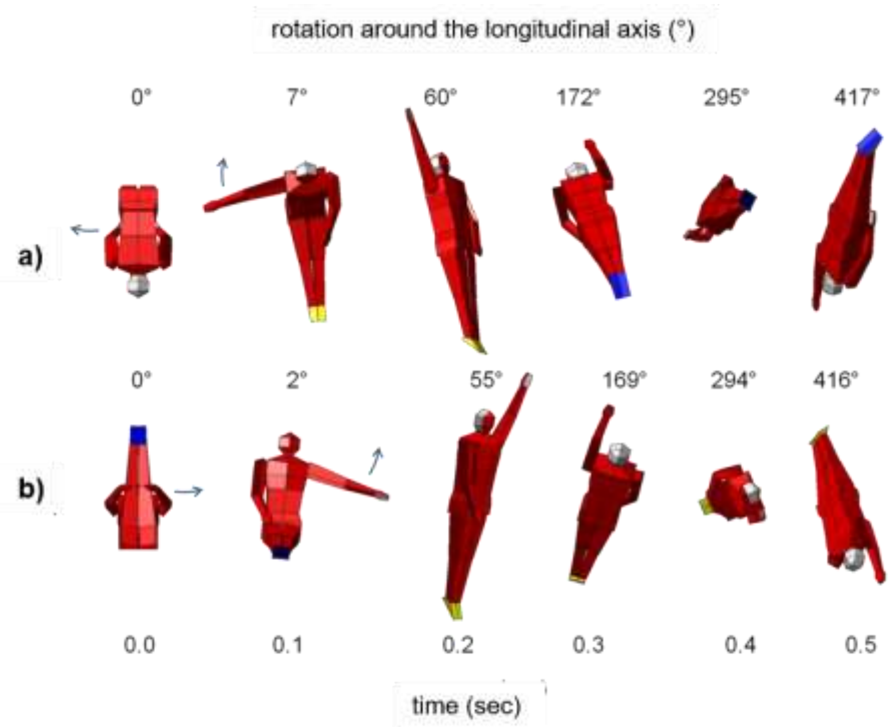

Fig. 3. 20 Initiation of a twist to the left through raising sideward (a) the right arm in back somersault превъртане назад; (б) left arm in front somersault превъртане напред, (view – to the front)

One very good possibility for initiating a tilt and provoking a twist of the body in stretching is observed when one of the arms is raised up to the sides of the body. Figure 3.20 shows movements of one arm up sideward – the right arm in basic element back pike somersault (a) or – the left arm in basic element pike somersault (b). Although the quantity of the twists is very similar in the two simulations, there are more beneficial conditions for additional increase of the lateral inclination (and of the longitudinal rotation) in the option back pike somersault.

Another posibility for provoking rotation around the longitudinal axis, in the transition from pike to straight somersault, is a sequence of several relatively easy to perform movements (fig. 3.24).

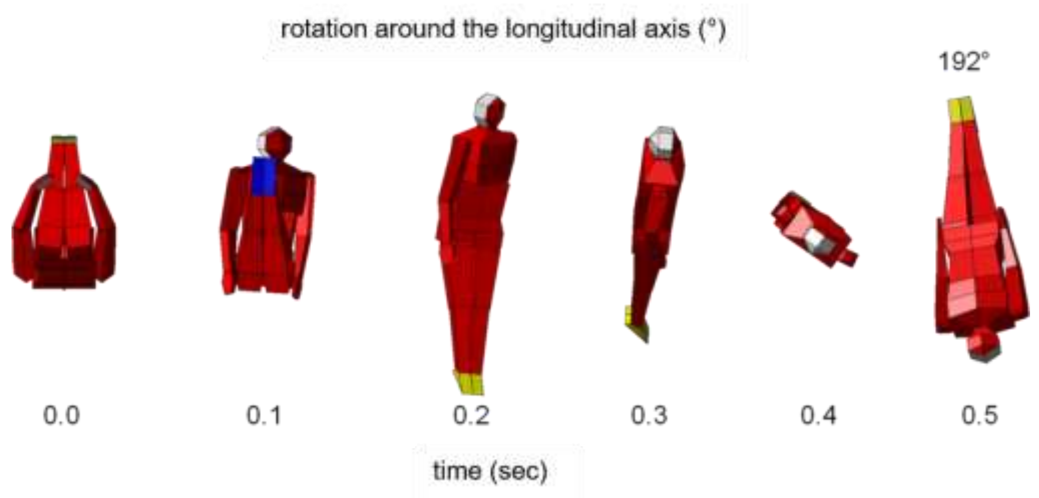

Fig. 3.24 Initiating a twist to the left by torsion the torso followed by stretching the body (in a pike somersault with forward rotation)

From the initial pike position of the body, the movements begin with a torsion in the upper part of the torso in the desired direction of rotation around the longitudinal axis (in this case to the left). Then, there is extension in the hip joints and then a torsion in the opposite direction (to the right) for recovering to the initial orientation between the upper and lower parts of the body. Practically, the execution of these movements is performed with a certain “overflow” between the different phases. As a result of the movements, in the final straight position, the body is inclined to some extent sideward (fig. 3.24, image 4).

The next movement shown (fig. 3.27) actually resembles a ¼ cone circle with the lower limbs and a curve in the area of the torso (hula movement). This techniqe is well explained in the surveys of Yeadon (1993в)[6]. Our numerical experiments also confirmed the practical realisation of this type of movements.

[6] Yeadon, M. R. (1993в). The biomechanics of twisting somersaults Part III: Aerial twist. *Journal of Sports Sciences*, 11, 209-218.

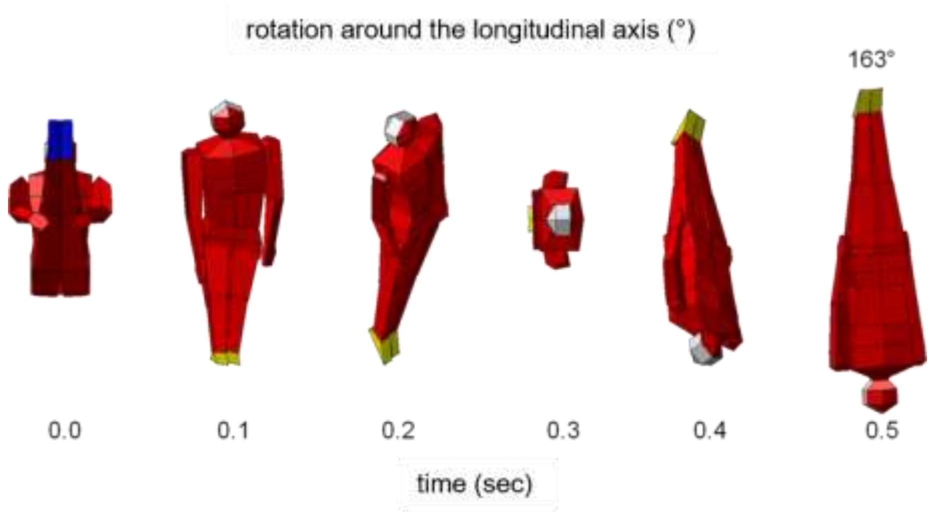

Fig. 3.27 Initiating a twist to the left by performing a lateral flexion to the right upon extension in the hip joints followed by stretching the body (in a pike somersault with forward rotation)

We should point out that this technique is suitable only when the pike somersault is with a forward rotation.

From the last three options viewed, we found that in a transition from pike to straight somersault, regardless of the direction of the rotation of the somersault, the greatest effect on the quantity of the twists is achieved when one of the arms is raised up to the side (fig. 3.20). The results confirmed the inappropriateness of application of the other two technical options when the somersaults rotation is backward.

3.1.3 Creating rotational motion of the body around the longitudinal axis during the support phase

When in a somersault, the twist is initiated at the end of the support phase, the vector of the angular momentum is at a certain angle (θ) in relation to the horizontal plane (fig. 3.28). The magnitude of this angle depends on the ratio of the rotation movements created around both the lateral and longitudinal axes of the body upon leaving the support.

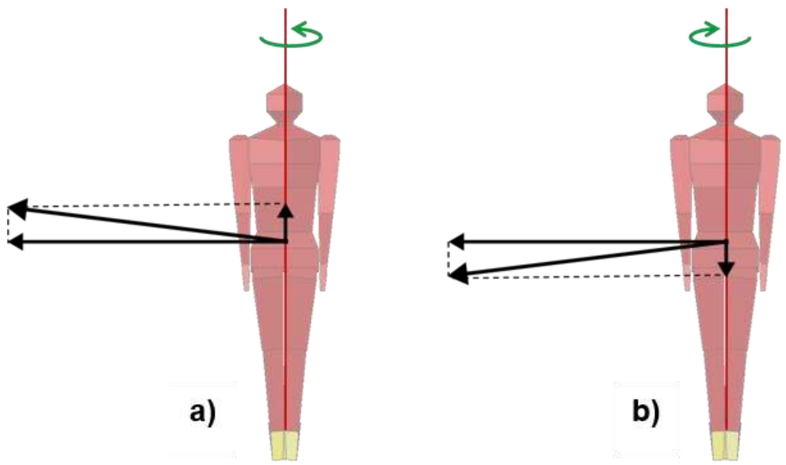

Fig. 3.28 Angular momentum (and components of angular momentum) in back straight somersault with created rotational motion around the longitudinal axis upon the contact with the support: (a) rotation to the left; (b) rotation to the right

Through simulations, we established the quntity of the increase in the angle of the tilt after ½ of the somersault with an increase in the quantity of the twists (fig. 3.30).

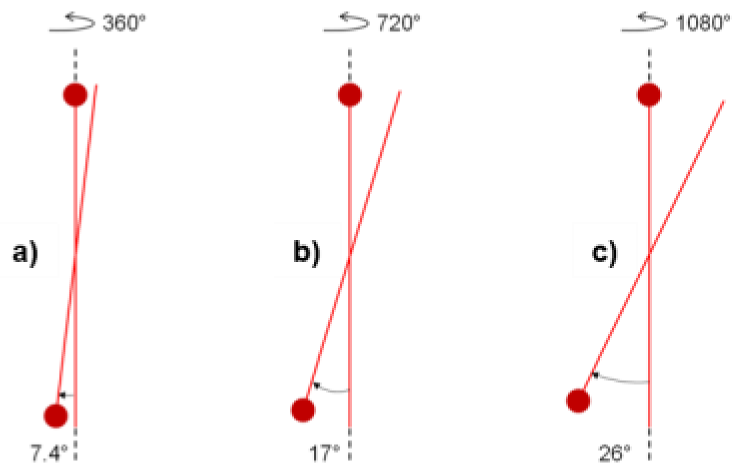

Fig. 3.30 Angle of the tilt after ½ of the back somersault depending on the quantity of the twists: (a) at 360°; (b) at 720°; (c) at 1080°

In the combination of a somersault and a twist, the variations (nutation) of the magnitude of the angle which is made by the principal longitudinal axis of inertia of the body and the plane, perpendicular to the vector of the angular momentum, can be successfully applied in solving a number of motor tasks. This angle, in the course of the movement, increases with every odd ¼ rotation around the longitudinal axis (90°, 270°, etc.) and regains its initial value with every even ¼ rotation (180°, 360°, etc.).

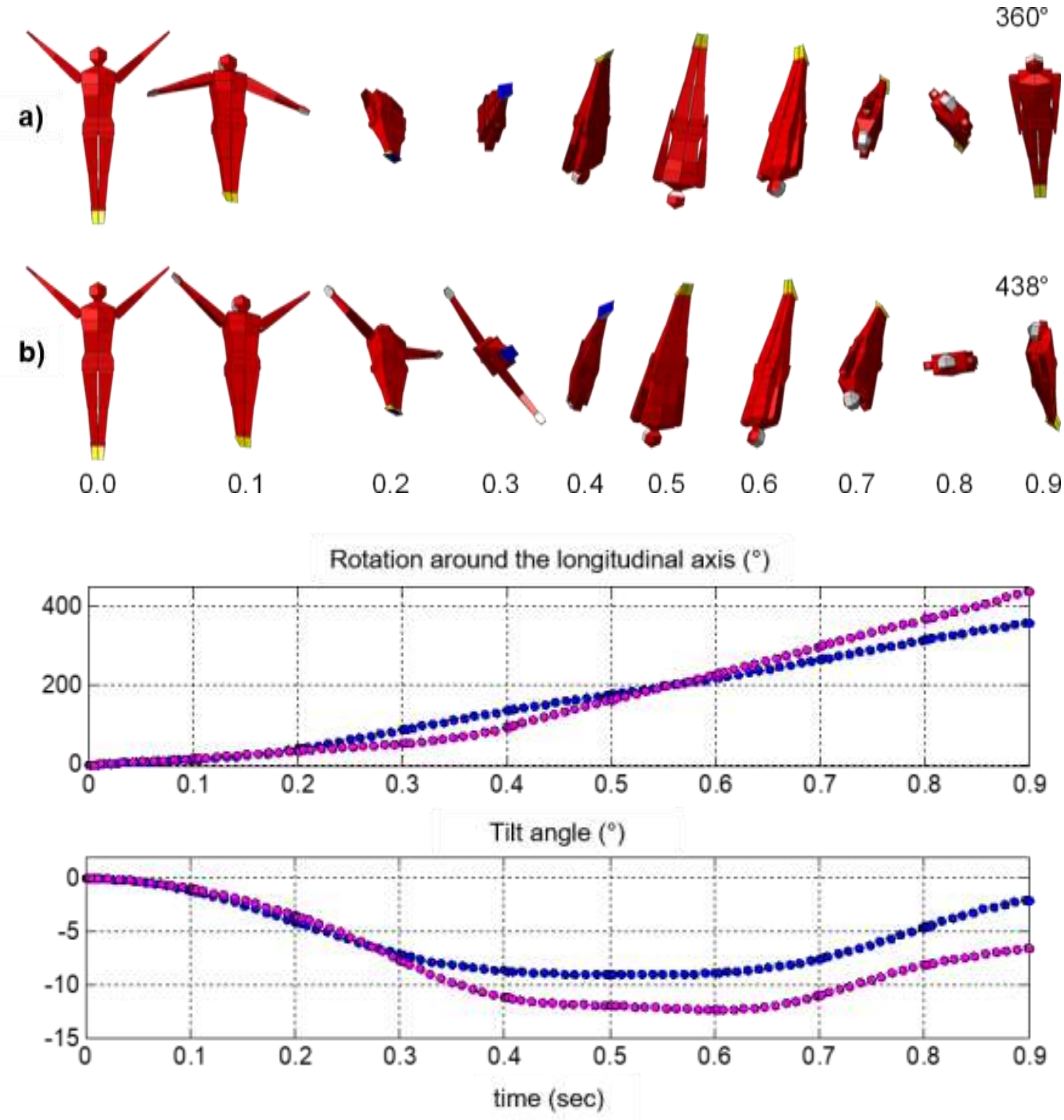

Fig. 3.32 Quantity of twists and tilt depending on the moment of lowering the arms in back straight somersault: (a) and (●) – the arms are lowered in the beginning; (b) and (●) – the arms are lowered after 0.2 sec

As Yeadon explained (1993б)[7], in initial position of the arms to the sides, the variations of the angle are larger. When performing a kind of a somersault and rotary impulse around the longitudinal axis provided by the support, if we begin to lower the arms not immediately but around the moment of reaching up to ¼ rotation around the longitudinal axis, i.e., around the moment when the tilt is maximum, the body will begin to rotate around the longitudinal axis with a greater angular velocity. Since this technique is based on the effect from nutation, Yeadon called it nutation technique.

[7] Yeadon, M. R. (1993б). The biomechanics of twisting somersaults Part II: Contact twist. *Journal of Sports Sciences*, 11, 199-208.

3.1.4 Termination of the rotation around the longitudinal axis and preparation for landing

The importance of landing for forming the impression from the overall execution of the elements is indisputable. The successful solving of motor tasks related to the realization of the rotations can be discredited by the inaccuracy of the landing.

We will focus on two major problems arising in the movements with combinations of rotations. The quality of the landing depends on their successful overcoming. One of the problems is the tilt which exists objectively and can lead to both unstable landing and injuries. The other problem is the presence of a certain extent of residual quantity of rotary motion around the longitudinal axis in the final phase which is also a premise for unsuccessful landing or injuries.

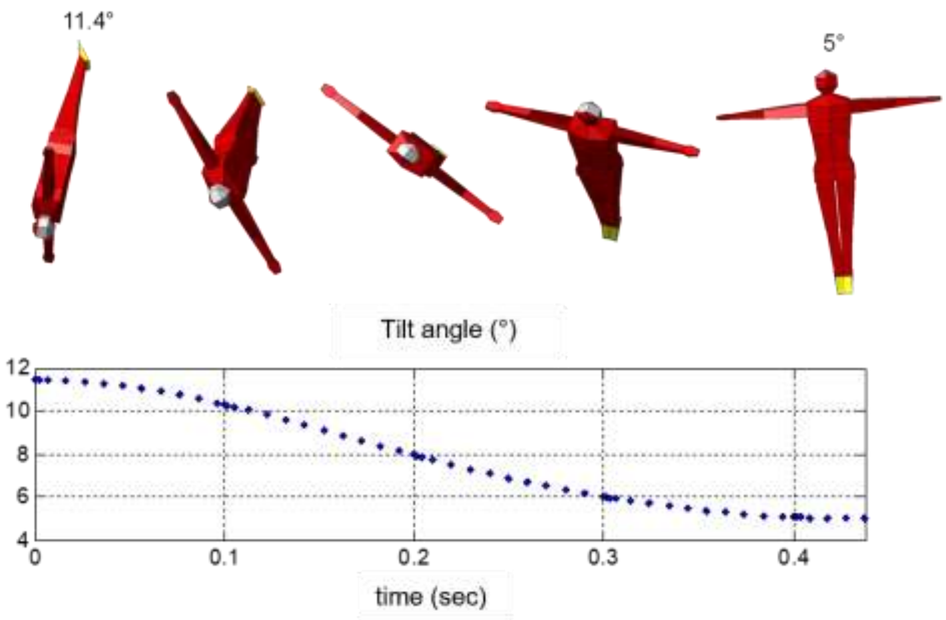

Fig. 3.33 Reducing the magnitude of the lateral inclination of the longitudinal axis of the body through applying a reverse nutation effect

It turned out that we can take advantage of the nutation effect not only for additional lateral inclination but also for reducing the lateral inclination of the longitudinal axis. The lateral inclination can be reduced in certain limits if the arms are raised the moment when there is ¼ of the longitudinal rotation left until the end of the movement (fig. 3.33). When completing the rotation around the longitudinal axis with arms to the sides, the angle of the lateral inclination decreases. We will call this effect “reverse nutation effect”.

First, we direct our attention to movements with combinations of rotations where the landing faces the floor. These are all kinds of back somersaults with quantity of twists with rate frequency of a complete rotation, i.e., 360°, 720°, 1080° and forward somersaults with quantity of twists with rate frequency of ½ uneven number of rotations, i.e., 180°, 540°, 900°.

We apply a technical solution which we called a counter nutation affect, but with raising only one of the arms to the side. This reduces the tilt, but for greater efficiency we apply additional actions and form the so-called special technique in landing facing the floor which ensures better final landing conditions (fig. 3.34b).

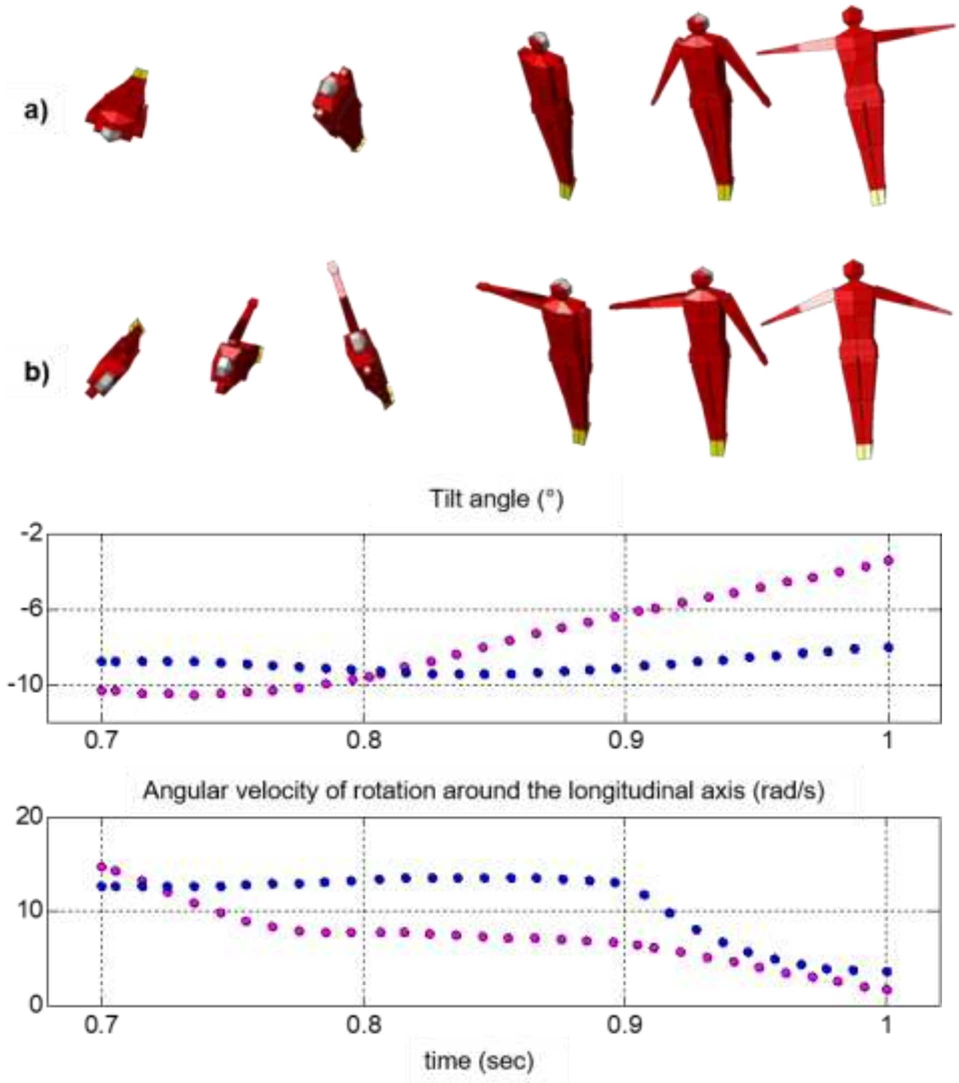

Fig. 3.34 Preparation for landing facing the floor through:
(a) and (•) – raising the arms to the sides at the end of the phase;
(b) and (•) – applying special technique

In the other option of landing, in the final part of the non-support phase of execution of gymnastics elements, the body is with its back to the floor. This position is normal in the execution of forward somersaults with quantity of twists in rate frequency of a complete rotation (360°, 720°, 1080°, etc.) and back somersaults with quantity of twists in rate frequency of ½ uneven number of rotations, i.e., 180°, 540°, 900°, etc.

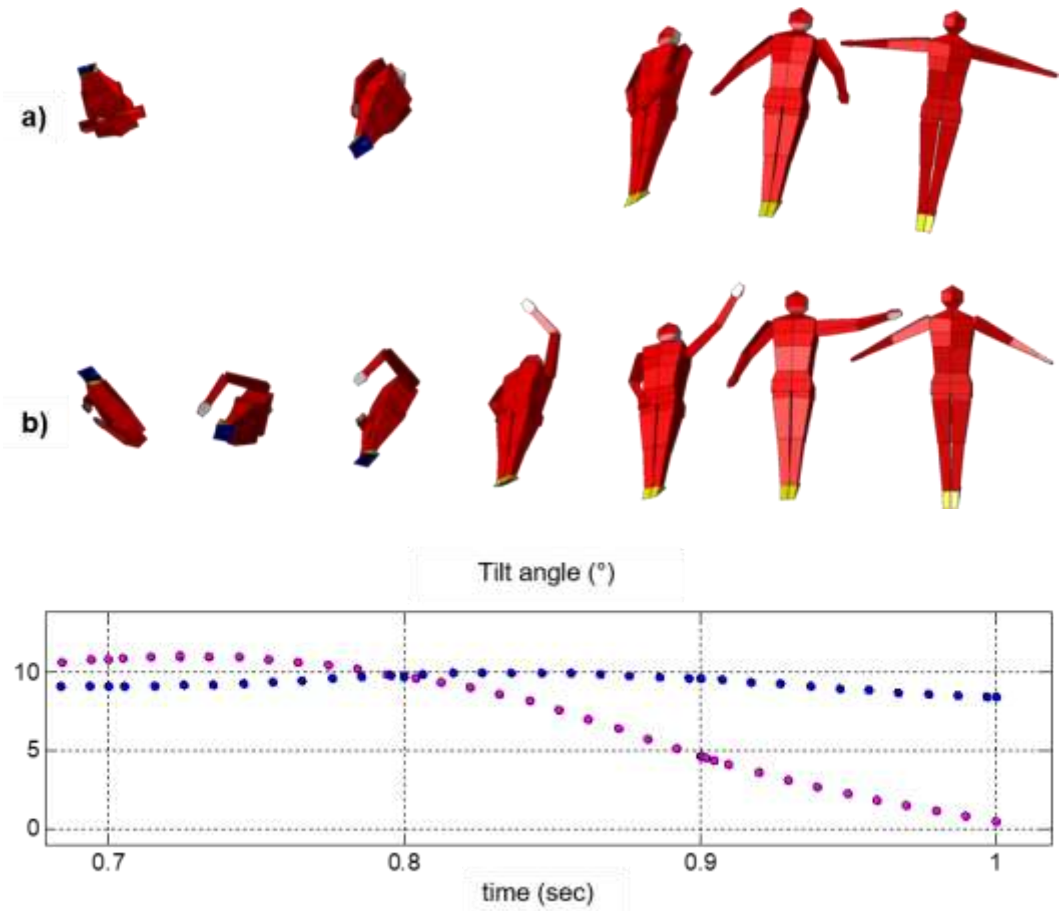

Fig. 3.35 Preparation for landing - the body is with its back to the floor through: (a) and (●) – raising the arms to the sides at the end of the phase; (b) and (●) – applying special technique

The preparatory phase for landing, during which the special actions are performed (fig. 3.35b), begins the moment when there is approximately ¼ more rotation of the body around the longitudinal axis until the end of the exercise. In the beginning, the homonymous of the direction of longitudinal rotation of the body arm is raised forward. The arm is flexed at the elbow joint and is raised in front of the chest. This movement of the arm leads to a counter rotation of the body and the tilt is reduced. At the end of the first cycle of movements, the arm is directed upward and then sideward being stretched at the same time. In the second cycle both arms move simultaneously, and the arm which begins the movement first is lowered to the side approximately to horizontal position, and the other arm is raised to the side to approximately horizontal position (fig. 3.35b).

After the analysis of the results from the numerical experiments, we arrived at the conclusion that regardless of the way of creating the rotational motion around the longitudinal axis of the body (support or non-support option), the proposed special technical actions can be successfully applied for increasing stability and security of the landing.

3.2 Biomechanical profile of the gymnastics elements with non-support phases

The transition from execution of easier motor actions needed for performing a smaller quantity of longitudinal rotation to actions aimed at providing maximal quantity of longitudinal rotation can be facilitated if there is continuity and similarity in the rhythm between the actions of the different stages. When learning the elements taken from the initial levels of difficulty, we encounter the question about which of the possible applicable basic technical options possesses a suitable rhythmical basis and technique which can be cognate to the technique applied in the most difficult options of a particular exercise. To find the answer to this question, we apply an approach which technologically breaks down to the following: first, through purposeful simulations we build the movements by whose execution we reach the toughest option, i.e., the option with maximum quantity of longitudinal rotation. Then, we reduce the difficulty by gradually decreasing the rotation productivity of the actions. This is expressed in: reducing the amplitude and speed of the actions; decreasing the chronological precision and increasing coordination accessibility of the actions; rising the time span for manifestation of certain activity. When building the difficulty options from a certain chain, we always take into consideration the preservation of the homogeneity of the rhythm of the movements. After all, for every kind of exercise, we find this easiest (e.g., with longitudinal rotation of 360°) option of execution and we already know that it can be made maximally complicated successfully and with relatively available actions.

Each model is presented as a selected sequence of images which clearly illustrate the movements and the technology of the actions in the course of the execution. The images are accompanied with explanations, analysis, and reasons for applications of the actions.

We created optimized options of 51 elements on different gymnastics apparatus; 115 trials of the elements; 47 experimental movements. We will present the model executions only of elements for floor gymnastics of the kind double back straight somersault with twists (non-support option).

3.2.1 Floor exercise (non-support option)

In the beginning, for every profile of elements we introduce a model of a basic element presented with characteristics which have to be achieved in order to proceed with learning the more complicated options of the element in the offered sequence of motor programs for the different levels of difficulty.

Basic double back straight somersault

In fig. 3.38 we can clearly see the orientation and configuration of the body, as well as the movements of the different segments, performed in intervals of 0.1 sec.

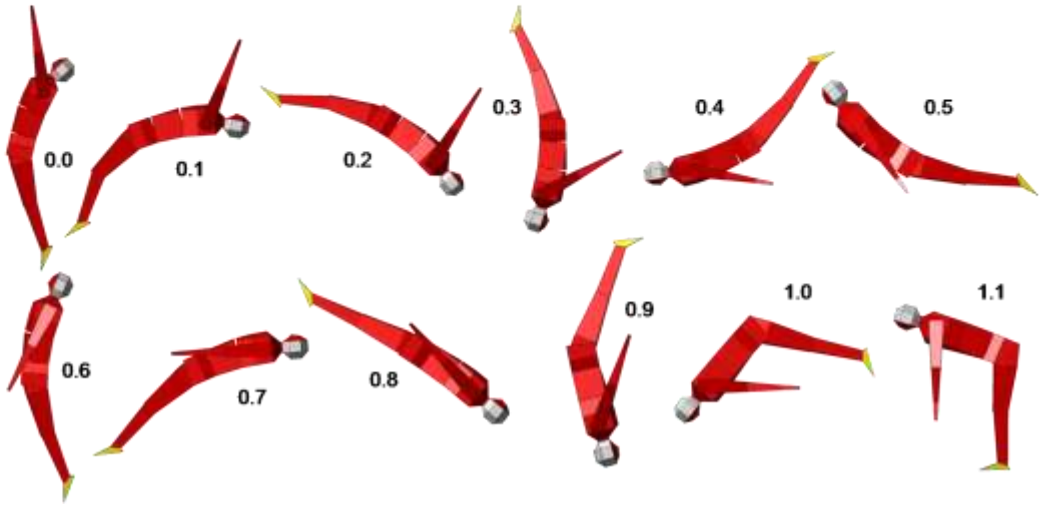

Fig. 3.38 Model execution of double back straight somersault

If the gymnasts succeeds in learning the element in a way identical with the one shown in the figure, he will have the potential for making the element more complicated with a twist. The angular momentum (norm.) is presented as a rotation potential needed for this basic execution and has a value equal to 1.847. The presented data can be used as reference points when learning the basic level of the element.

Double back straight somersault with a twist - 1080°

In this case, it is important that the motor actions be directed not towards an immediate longitudinal rotation but towards achieving a configuration where the arms can perform productive actions for creating a great lateral inclination.

Figure 3.39 shows a model execution of double back straight somersault with a twist of 1080°.

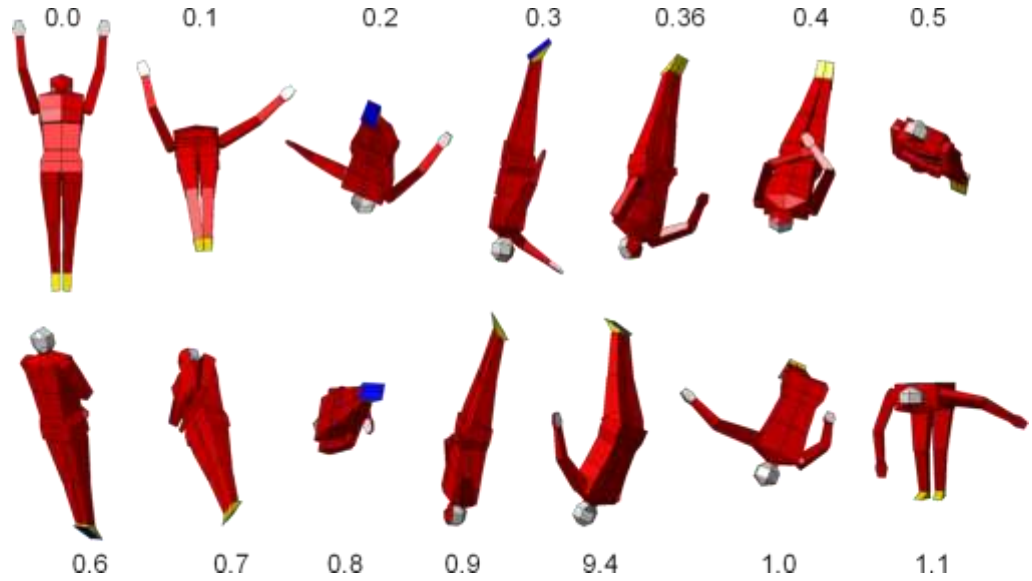

Fig. 3.39 Model execution of double back straight somersault with a twist - 1080°, (view – to the front)

The next figure 3.40 shows only the phase of initiation of rotation around the longitudinal axis. The rotation movement of the somersault is not presented for achieving a more convenient observation and focusing only on tilt actions. We can see in the figures that lowering the arms starts at the beginning of the non-support phase to help rotation of the somersault. The homonymous of the direction of longitudinal rotation arm (in this case the left one) is lowered to a greater extent to the side, and the right arm is lowered to the front with a minimum movement sideward. But the arms are moved in a way so that they do not lead to a longitudinal rotation of the body to the left (there is even a minimum counter rotation of - 4° to the right, image 2, t = 0.1 sec). This is achieved due to the fact that the left arm when moved forward is also moved sideward, i.e., to the left, which leads to a minimum longitudinal rotation of the body to the right and despite the provoked minimum sideward inclination, the longitudinal rotation to the left is still missing (image 2, t = 0.1 sec).

Fig. 3.40 Model execution of phase initiation of rotation around longitudinal axis - 1080°, in double back straight somersault, (view – to the front). The rotation movement of the somersault is not shown

The premature rotation to the left will deprive the competitor from the possibility to use maximum the movements of the arms to initiate a tilt. From this position (t = 0.1 sec) very active actions for creating a great quantity of lateral inclination begin. The left arm continues its movement down to the side, and the right arm is moved sidewards up. In this rotary motion of the arms in a clockwise direction, the body counter rotates (tilts) in the opposite direction and consequently, it starts rotating around the longitudinal axis to the left (image 2). In order to achieve an additional tilt, the movement of the arms is performed together with a lateral flexion of the torso (in sum about 10°, in the greatest extent between t = 0.2 and t = 0.3). After reaching about ¼ to ½ of the longitudinal rotation, the body is stretched (from t = 0.3 to t = 0.4), which contributes to a new minimal additional tilt. The movement of the right arm plays a significant role for the magnitude of the tilt. After t = 0.2 (image 3), when the moment for reaching about ¼ of the longitudinal rotation approaches, the right arm is raised and is in a very suitable position for creating an additional tilt when lowered to the front. Upon lowering the arm, however, the body continues to rotate to the left and that is why, for a greater efficiency of the movement, the arm is moved slightly to the side (t = 0.36). Here, we want the movement to be performed in the plane which is perpendicular to the plane of somersault. Upon approaching ½ of the longitudinal rotation (between t = 0.36 and t = 0.4), the arm is moved sidewards towards the body for another minimal addition to the magnitude of the tilt. After this moment, both arms are close to the sides of the body and the movement goes into the phase of the realization of the longitudinal rotation. Here, due to the great values of the tilt (a little above 12° at 360° longitudinal rotation), we can achieve a very high mean value of the angular velocity of the longitudinal rotation for this phase (28.3 rad/s). Towards the end of the movement in the preparatory phase for landing, we apply the above-mentioned technique for eliminating the tilt and terminating the twist until the moment of touching the landing surface. A little while before the position, when there is only ¼ of the longitudinal rotation to be performed till the end of the element (t = 0.9), we begin to raise the right arm to the side for provoking a reverse nutation effect. Due to the great tilt in this element, at the end of the phase, the arms move actively for a maximum nautralization of the tilt by rotating in a counter-clock direction (between t = 1.0 and t = 1.1). Their assimetrical final position in this case is inevitable, but because of that, despite the extreme difficulty of this element, the body lands relatively comfortably and the lower limbs touch the surface without any visible tilt, and the angular velocity of the longitudinal rotation is reduced almost to zero.

Double back straight somersaults with a twist - 720°

By following the adopted approach for reducing the difficulty and increasing the level of availability and sticking to the requirement for similarity of actions, we designed another element – double back straight somersaults with a twist of 720° (fig. 3.41).

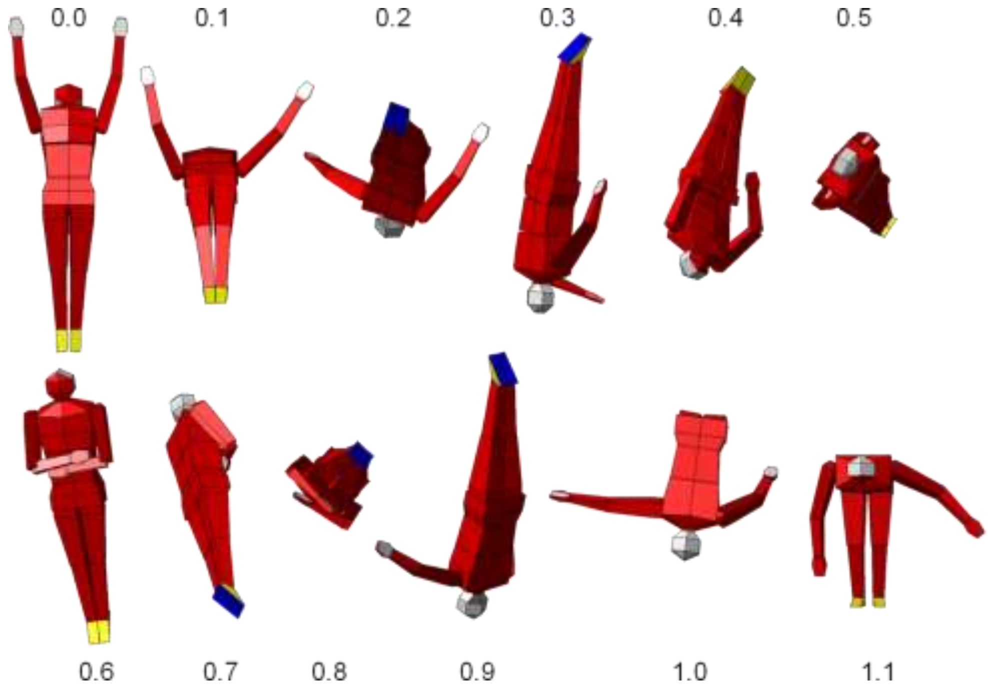

Fig. 3.41 Model execution of double back straight somersaults with a twist - 720°, (view – to the front)

Restraining from details, we can see that in this case, the technique is built on the same, already described, motor peculiarities of the execution. The difference in the movements lies in the smaller intensity and amplitude in their execution. During the phase of realization of the longitudinal rotation, compared to the previous execution, less tilt is achieved (around 8.7° at 360° longitudinal rotation), hence – a smaller magnitude of the angular velocity of the longitudinal rotation (20.1 rad/s). The smaller difficulty (in comparison to the previous element) facilitates, to some extent, the actions in the final phase – preparation for landing (fig. 3.41). Despite that, the magnitude of the achieved tilt is significant and after the compensatory movements the arms are still in an assymetrical final position.

Double back straight somersaults with a twist - 360°

We reach the easiest option for execution – double back straight somersaults with a twist - 360° (fig. 3.42).

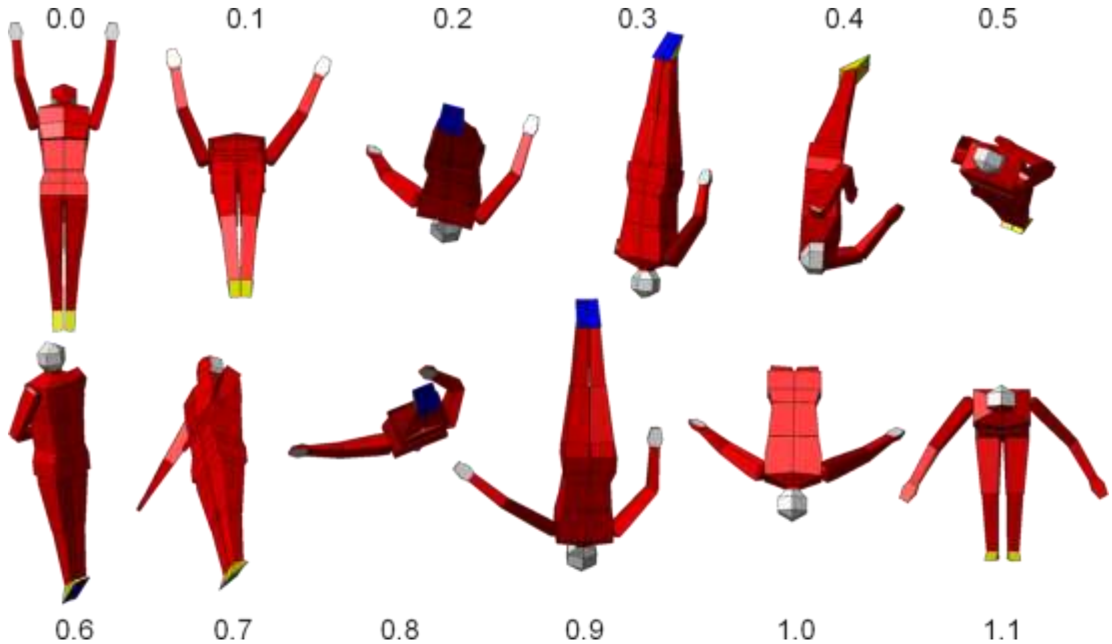

Fig. 3.42 Model execution of double back straight somersaults with a twist - 360°, (view – to the front)

Besides the expected reduction in the speed and the amplitude of the movements, here, we can see that the phase of the realization of the longitudinal rotation practically comes down to a momentary holding the arms to the sides around t = 0.5 and t = 0.6. Due to the small quantity (360°), the twist is performed mainly during the rest phases of the element (initiation and termination of the twist). The achieved tilt of about 5.7° and the not big angular velocity of the longitudinal rotation of 12.4 rad/s after applying the special technique, are easily reduced to zero values which ensures a trouble-free landing. In this easy option, the arm movements during the preparatory phase for landing are expressed and graceful and remain simmetrically positioned at the end of the movement.

Similarity in the actions for initiating a twist in the different level of difficulty (in double back straight somersault)

Although in the overall executions, the movements seem different to a certain extent, if we direct our attention only to the relative movements (fig. 3.43), we find that the actions in the different options are extremely similar.

We will outline the trends in the differences in direction from the easier to the more difficult option of execution. When we make the element more difficult, the left arm is lowered a little more to the front and is moved more to the side (t = 0.1 and t = 0.2). The right arm is also lowered to a little greater extent (until t = 0.1), then, it changes its movement by going to a greater extent to the side and up (t = 0.2 and t = 0.3). This is followed by a more active lowering to the front and a little more to the side for the more difficult options (t = 0.4), which enables an additional sideward movement of the arm towards the body at the end of the phase (t = 0.5). The maximal lateral flexion of the torso is also significantly big (t = 0.3) in the more difficult options, and in these options, in order to reduce the inertia momentum at the end of the phase, the arms are close to the sides of the body. As a whole, the movements of the arms (and the torso) are characterized with a greater scope and speed when making an element more complicated. In the easier option, a greater flexion in the arms is permissible, which slightly facilitates their movement.

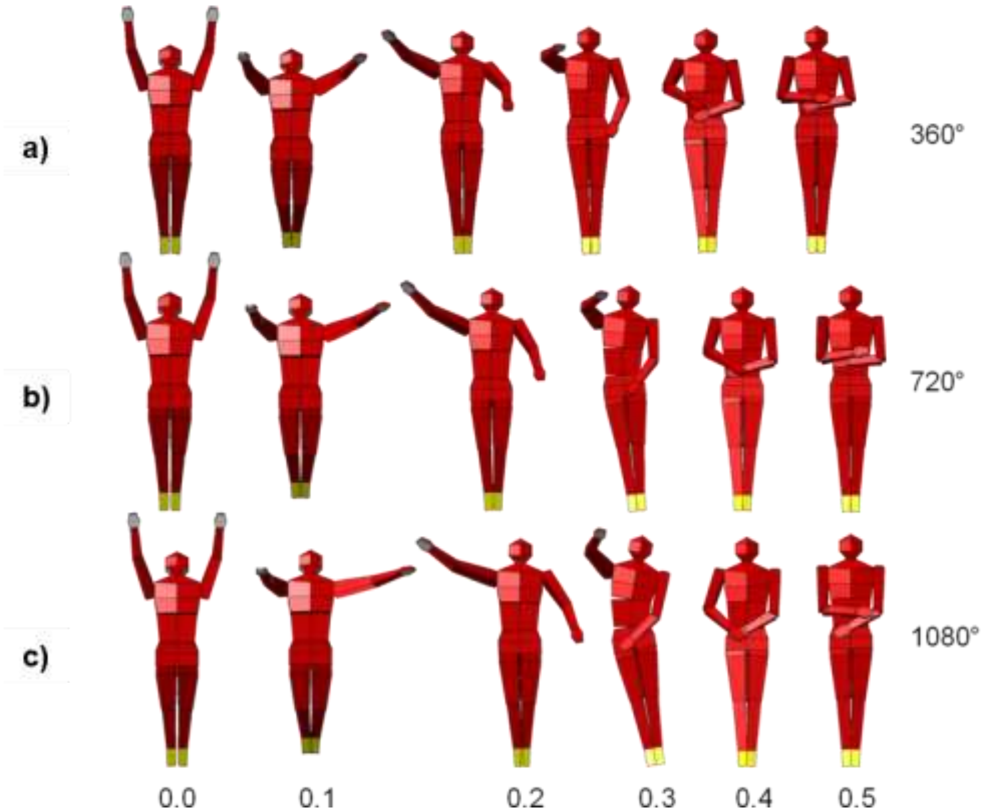

Fig. 3.43 Comparison between the relative actions during the phase of initiation of a rotation around the longitudinal axis in double back straight somersaults with a twist: (a) 360°; (b) 720°; (c) 1080°

We can sum up that there is a great degree of similarity between the relative movements in the above-mentioned element, which is a premise for the successful learning of the more difficult options after learning the basic easier option. We should point out that there is a strong bilateral dependence between the movement of the arms (and torso) and the transitional longitudinal rotational motion. Obviously, the efforts in the educational process should be directed towards achieving a maximum correspondence between the relative movements and the current longitudinal orientation of the body.

Conclusion

After discussing a number of elements with non-support phase (51 elements), we can look for a motor similarity in gymnastics actions in identical elements executed on different apparatus (e.g., more than one straight or semi-tuck somersault with a twist of 720°). From the elements with similar characteristics, the rotation potential of the somersaults is bigger in floor exercises. As for the initial position of the arms in relation to the torso, we found that he elements performed on floor and vaults ("Yurchenko" type) are the most similar. The position of the arms is the most distinct in rings dismounts. These peculiarities should be taken into accont when learning the elements on different apparatus. Regardless of the seeming visual differences, the goal of the initial actions in the different elements is the same. When the execution requires a bigger quantity of longitudinal rotation, the athlete's goal should be directed toward moving his arms fast to a position suitable for performing movements leading to a greater tilt of the body. This could be the position where the arms are approximately to the sides and a little to the front, and one of the arms is a little further to the front and up than the other one (e.g., fig. 3.91 image 1).

Fig. 3.91 Model execution of the phase initiation of rotation around the longitudinal axis (view – to the front). The rotational motion of the somersault is not presented

Until the moment of taking this key position of the arms, the body should not change its longitudinal orientation, i.e., the movement of the arms to this position should not lead to a longitudinal rotation of the body. Otherwise, the efficiency of the consequent actions will be reduced significantly. The experiments confirmed that a bigger quantity of the twists can be realized when the initial actions are directed not toward immediate, fast initiation of longitudinal rotation, but to taking a position which favors actions leading to a maximum tilt. This, of course, causes certain delay in the beginning of the longitudinal rotation, but the effect of this motor strategy is in providing a greater angular velocity of the twist, which eventually leads to a bigger quantity of longitudinal rotation of the body until the end of the executed element. Right after reaching the work position, the arms start performing a circular movement (one arm moves up and the other arm moves down) in the face plane which leads to a counter rotary rotation of the whole body in the opposite direction (in this case – tilt to the right). The circular movement with the arms can be performed without coordination difficulties and its effect is enhanced when performed together with a lateral flexion in the same direction in the area of the torso and waist (fig. 3.91 images 2 and 3). This circular movement of the arms leads to a tilt and the body starts rotating around the longitudinal axis. Afterwards, the efficiency of the circular movement of the arms decreases. This is due to the fact that as a result of the longitudinal rotation of the body, the plane where the arms move starts shifting from the plane perpendicular to the vertical plane of the somersault превъртането. The benefit from arm movements significantly decreases when we approach ¼ longitudinal rotation. At the end of the movement, however, one of the arms remains raised and its lowering to the front becomes quite efficient when the arm movement is performed in the plane perpendicular to the vertical plane of the somersault (images from 3 to 6). Lowering the arm leads to a counter rotation of the body, i.e., the magnitude of the tilt is increased. The productivity of arm lowering is depleted when we approach ½ longitudinal rotation. A final minimal additional tilt can be achieved when the body performs about ½ longitudinal rotation and the arm remains a little to the side at the end of its lowering (image 6) so that it can move sideward and provoke a minimal counter rotation of the body. In brief, the athletes should strive for moving their arms mostly in the plane perpendicular to the vertical plane of the somersault. This means that the speed and direction of the arm movement depend on the current longitudinal orientation of the body and the speed of its change. On the other hand, the speed of the longitudinal rotation is influenced by the actions of the arms, i.e., the direction and speed of movement of the arms and the angular velocity of the longitudinal rotation are completely interdependent. Because of that, in order to achieve a great efficiency in the actions, we should possess a significant technical mastery in the execution of the relative movements of the body segments, as well as precision in the evaluation of the behavior (orientation) of the body in space. The desired universality can be achieved significantly more easily and quickly when for the initiation of a longitudinal rotation we apply the non-support option which provides a similarity in the work actions and similarity in the sequence of their execution.

CONCLUSIONS AND RECOMMENDATIONS

Conclusions

1. We created a mathematical model for computer simulations of motor actions in the conditions of non-support phase. The model possesses a high degree of mobility which makes it suitable for surveying various movements and actions charcteristic for gymnastics elements. The mathematical model can be applied for research activities for movements with non-support phases in other sports as well.
2. We designed a computing program for conducting numerical experiments which is working in the MATLAB Computing Environment. It is easy to use and can be applied for: research of the effect of the execution of various motor actions; studies of details of sports technique; establishment of the efficiency of certain motor strategies; creation of new elements with flight phases.
3. Thanks to the integrated in the basic program additional program modules (visual and numerical), the results from the simulations can be presented in a clear and convenient for perception view, and the calculations of the numerous biomechanical characteristics contribute to the detailed analysis and facilitate the interpretation of the obtained results.
4. We found new and known facts related to body behaviour. We clarified objective mechanical effects in athletes' movements during the non-support phase which should be taken into account when seeking motor efficiency.
5. We established the actions of the motor sequence ensuring a high efficiency when a twist is initiated, as well as motor strategies providing conditions for stable and safe landing.
6. We established the motor actions possessing similar rhythmical basis and are present in the technical structure of the elements of one type – from the easiest option to the most complicated option of a certain element.
7. We built model executions of elements on different gymnastics apparatus where due to the similarity in the motor sequence, the more complicated options of each exercise can be learnt more easily and quickly on the basis of a positive transition of motor abilities. We made an archive of files (vectors of control and initial conditions) of 51 elements, 115 trials of the elements, 47 experimental movements, which can serve as a basis for future research.
8. In the elements with a greater rotation potential of the somersaults, the actions for initiation of twists are more efficient, but the requirements for their precision are higher. There is the biggest rotation potential of the

double straight somersaults двойните in the elements from floor exercise – 1.85, and the smallest – in the elements on rings – 1.42.

9. When initiating a twist during the non-support phase, there are better conditions for control of the movement and a possibility for achieving a bigger angual velocity (e.g., 28.3 rad/s for the non-support option and 23.7 rad/s for the support option of double back straight somersaults on floor), but in order to achieve a higher efficiency of the actions, we should possess motor precision and high technical skills.
10. In the support option, the actions leading to a longitudinal rotation can be considered technically available but they are deprived from sufficient universality and carry a risk of loss of efficiency of the actions prooviding the rotational motion and height of the somersaults. On the other hand, in this option there are potential problems in landing.

Recommendations

1. We recommend that sports pedagogues pay attention in the preparation to the basic elements. The presented mdel performances and the values of the necessary rotational potential can be used as a reference point for achieving a high quality of execution. Presented in a normalized kind, the values can be applied for athletes with different anthropometric characteristics and different duration of the flight phase.
2. When learning the elements with combinations of rotations, we recommend that the actions leading to initiation of twists be performed during the non-support phase (non-support option), and then, when the elements is made more complicated, the suggested model motor programs for the more difficult options of a certain elements be followed.
3. We recommend, during the different stages of education, that the technique for initiation of longitudinal rotation in non-support phase be perfected by being applied in exercises with different difficulty level (e.g., from a trampoline into a foam pit) in order to develop and maintain the abilities for control of the tilt and the speed of the longitudinal rotation.
4. In non-support initiation of a twist, if a big (maximum) quantity of twists is needed, we recommend that the actions be directed not towards provoking an immediate longitudinal rotation, but towards a fast creation of conditions for initiation of maximum tilt which consequently will ensure a high angular velocity for longitudinal rotation.
5. In the complicated options of the elements, we can apply some actions for initiation of a rotational impulse (not big) around the longitudinal axis right at the end of the support phase (support option), and then create suitable conditions for providing an additional impulse from the actions performed

during the non-support phase. In some elements, we can exploit the so-called nutation effect to increase the magnitude of the tilt.

6. As for floor gymnastics, we recommend the application of the so-called support technique in the execution of single straight somersaults with a twist bigger than 360°.
7. For providing a successful landing (stable and without any risk of injuries), we recommend applying the introduced motor strategies for the elements where in the landing the body faces the floor or is with its back to the floor.

Contributions of the dissertation (in the author's opinion)

1. We created a mathematical model for computer simulations in the conditions of non-support position with high degree of mobility allows for performing cognitive activities on various motor activities. The model proved its efficiency after being applied in the survey of different gymnastics elements.
2. We designed a computer program for control of the simulations, the behaviour of the model, and calculation of the biomechanical characteristics whose management is intuitive and convenient.
3. We established a highly productive options for the technique for initiation of twists based on the similarity of the motor sequence, which would facilitate learning the more complicated options of the elements of a certain type.
4. We developed optimized technical solutions and created a file archive of 51 elements, 47 experimental movements with their initial conditions.
5. We developed motor strategies for facilitating the landing in elements with combinations of rotations which provide a better stability and decrease the risk of injuries.

Publications related to the dissertation

1. Кючуков, Ил. (2006). Равнинен модел за изследване на движението при безопорната фаза. *Спорт & наука*, (3), 14-22. Kyuchukov, Il. (2006). Plane model for surveying the movement in non-support phase. *Sport & Science*, (3), 14-22.
2. Кючуков, Ил. (2006). Изследване на ориентацията на тялото на спортиста в безопорната фаза на гимнастически упражнения при равнинно движение. *Спорт & наука*, извънреден бр., (1), 18-25. Kyuchukov, Il. (2006). Research of the orientation of athlete's body in non-support phase in gymnastics exercises with plane movement. *Sport & Science*, special edition, (1), 18-25.

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4. Кючуков, Ил. (2008). Изследване на ротационння потенциал на тялото на спортиста при упражнения на прескок. *Спорт & наука*, извънреден бр., (3), 29-36. Kyuchukov, Il. (2008). Research of the rotation potential of athlete’s body in vault exercises. *Sport & Science*, special edition, (3), 29-36.